Overview of Nuclear Reactor Physics and Kinetics

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Introduction (5/5)

Contents of lecture

0. Introduction

1. Derivation of reactor kinetics equation

2. Reactor response for a simple reactivity change by solving the reactor kinetics equation

Recommended book

Weston M. Stacey "Nuclear Reactor Physics, Second Edition"
2007 WILEY-VCH Verlag GmbH & Co. KGaA, Weinheim, ISBN 978-3-527-40679-1

Introduction (1/5)

The reactor kinetics is the characteristics of time-dependent behavior of neutrons in the reactor and it is one of the areas in the reactor physics.

■ One-energy-group neutron diffusion equation

Critical and steady-state reactor :

$$D\nabla^2\phi(\mathbf{r},t) - \Sigma_a\,\phi(\mathbf{r},t) + \nu\Sigma_f\,\phi(\mathbf{r},t) = 0\,. \qquad (1)$$

neutron loss rate

neutron production rate

$$\frac{d\,n(\mathbf{r},t)}{d\,t}$$

$$\left(\,\phi(\mathbf{r},t) = v\,n(\mathbf{r},t)\,\right)$$

Time-dependent case (which is not critical) :

$$\frac{1}{v}\,\frac{d\,\phi(\mathbf{r},t)}{d\,t} = D\nabla^2\phi(\mathbf{r},t) - \Sigma_a\,\phi(\mathbf{r},t) + \nu\Sigma_f\,\phi(\mathbf{r},t) \qquad (2)$$

For the time-dependent behavior of reactor, we can analyze it by directly solving the time-dependent diffusion equation. But it needs **so much costs**.

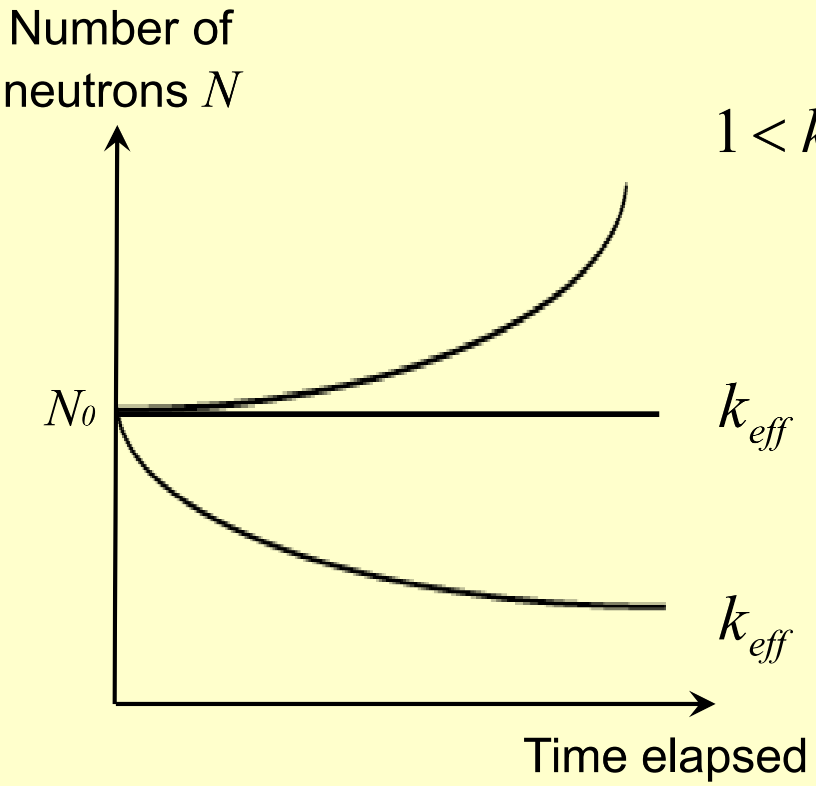
The reactor kinetics equation, that has a simple structure, is derived from the diffusion equation. And the analysis of time dependent reactor is usually carried out by solving this kinetics equation.

Introduction (2/5)

■ Effective multiplication factor

$$k_{eff} = \frac{\text{Neutron production rate by fission}}{\begin{array}{c}\text{Neutron production rate by fission}\\ \text{in previous generation}\end{array}} = \frac{\text{Neutron production rate}}{\begin{array}{c}\text{Neutron loss rate by}\\ \text{absorption or leakage}\end{array}} \quad (3)$$

All the neutrons produced are finally lost
(by absorption or leakage)



$1 < k_{eff}$ **Super criticality**
The number of neutron increases.

$k_{eff} = 1$ **Criticality**
The number of neutron is constant.

$k_{eff} < 1$ **Sub criticality**
The number of neutron decreases.
(with an exception)

Introduction (3/5)

■ Reactivity

$$\rho \equiv \frac{k_{eff} - 1}{k_{eff}} \quad (\Delta k / k) \qquad (4)$$

$$\begin{cases} \rho = 0 & \text{criticality} \\ \rho > 0 & \text{super criticality} \\ \rho < 0 & \text{sub criticality} \end{cases}$$

Relative deviation of k_{eff} from unity

More useful for use in the reactor kinetics than k_{eff}

Reactivity worth : difference of reactivity $\Delta\rho = \rho_2 - \rho_1$

The value of reactivity(or reactivity worth) is usually small, so many kinds of unit are used.

pcm 10^{-5} $\Delta k / k$

dollar

cent

(will be discussed later)

Example)

The reactor power is usually increased by adjusting the control rod with the k_{eff} being close to unity.

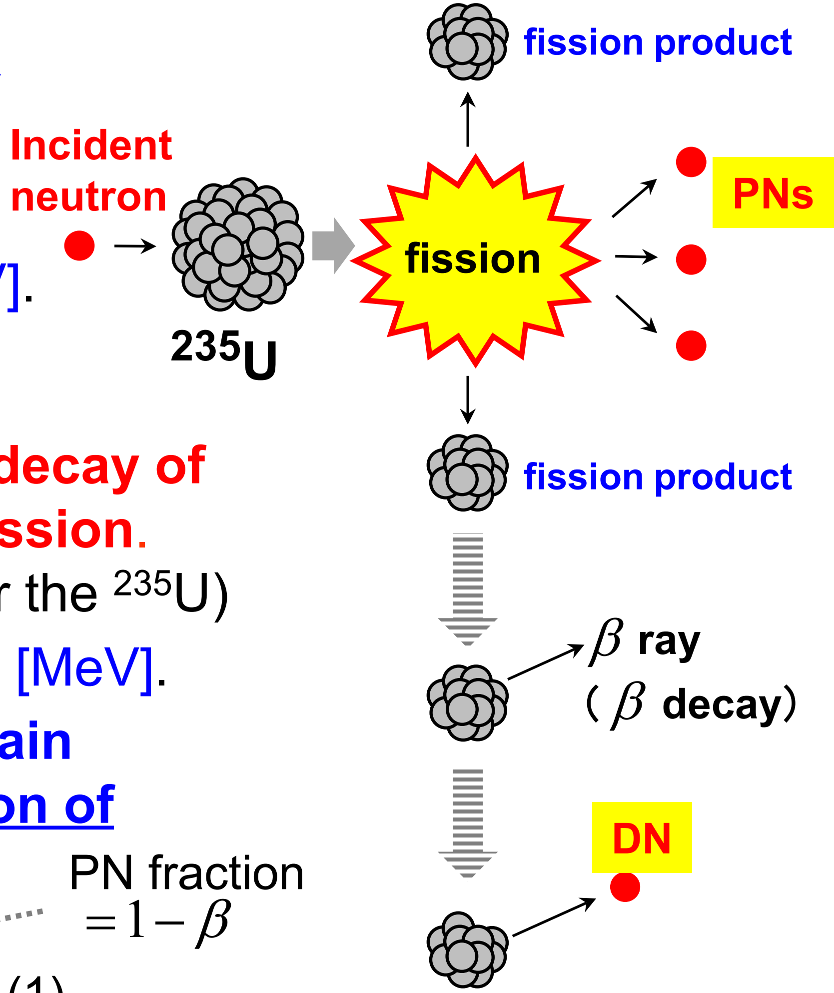
$k_{eff} = 1.0012$ ➡ ρ = 0.12 (% $\Delta k / k$) or 120 (pcm)

Introduction (4/5)

- The neutron number ∝ the fission rate ∝ the reactor power
- When the neutron number increases by a factor of 2, the reactor power increases by the same factor.
- This change is caused by a reactivity change introduced into a steady-state operating critical reactor.
- The reactivity changes are of two types : long-term and short-term.
- The long-term (months to years) changes are due to such effects as fuel depletion(consumption).
- The short-term (seconds to hours) changes are caused by control rod motions, reactor temperature change, etc.
- The short-term time-dependent behavior is generally classed as the reactor kinetics and that is the subject which we learn in this lecture.

1. Derivation of reactor kinetics equation

Prompt and delayed neutrons



Prompt neutron (PN)

2〜3 neutrons are usually emitted **promptly** at the fission. (mean number about 2.4 per fission for the ^{235}U)

Average energy at emission is about 2 [MeV].

Delayed neutron (DN)

Small number of neutrons are emitted **with decay of some of fission products long after the fission**. (mean number about 1.6 per 100 fissions for the ^{235}U)

Average energy at the emission is about 0.5 [MeV].

Both of PNs and DNs contribute to maintain the chain reaction during normal operation of reactor.

$$\text{DN fraction } \beta \equiv \frac{\text{Nbr. of DNs}}{\text{Nbr. of PNs + DNs}} \qquad (1)$$

PN fraction $= 1 - \beta$

(Ex. 0.65% for the ^{235}U)

Although the fraction of DNs is small, **their delay in the emission has a great impact on the reactor kinetics**.

Delayed neutron (1/3)

■ Process of DN emission

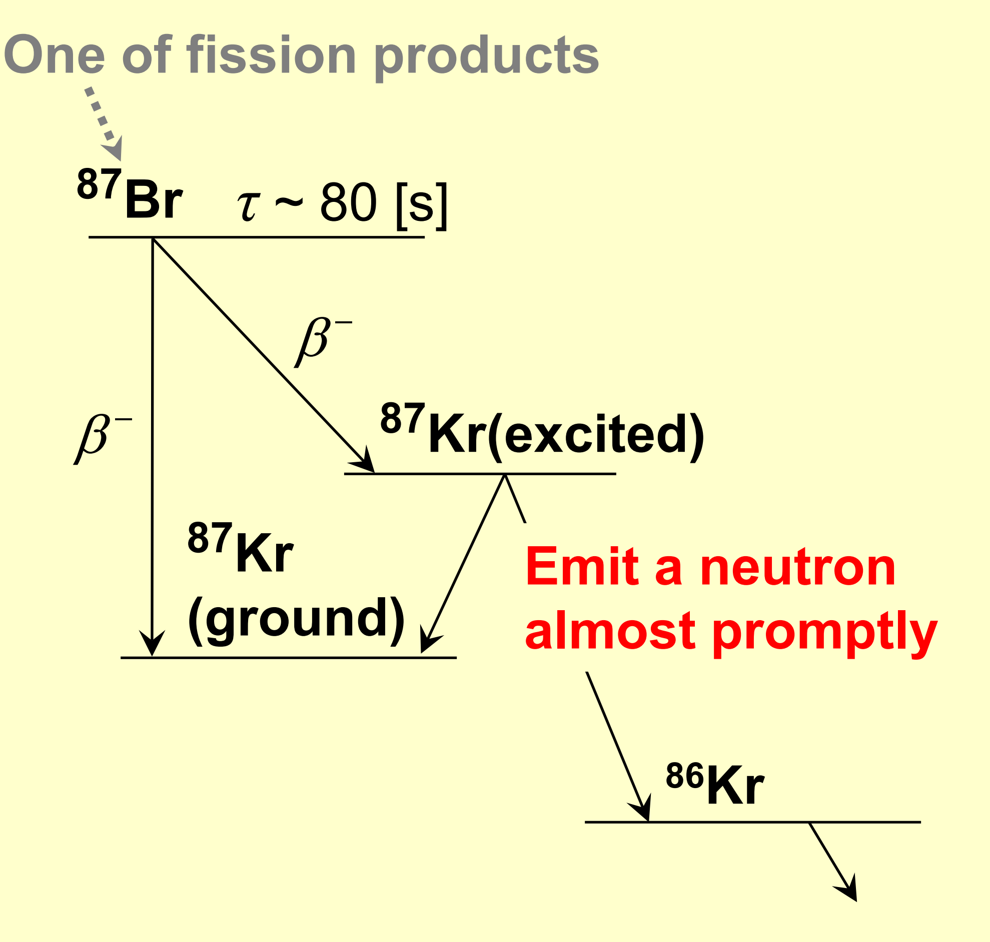
Many kinds of fission products have been found.
Most of them is unstable (“neutron rich”) and decays, emitting β rays.
Some of them also decays, **emitting a neutron**.

Example)

The life τ of ^{87}Br (80 [s]) practically determines the delay of the DN emission.

Delayed neutron precursor

A fission product like the ^{87}Br. It is known that about 50 fission products play as the precursor.



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life τ = half life / 0.693

Delayed neutron (2/3)

Six-DN-group data

It is impractical to treat 50 kinds of the precursors "directly" in the reactor kinetics.

Six-DN-group model is generally adopted.

Each group is composed of precursors that have a similar half-lives.

Table 1 Principal DN precursors classed into six groups

Principal precursors	Life [s]	Group
^{87}Br	80	1
^{137}I ^{88}Br	35.5 23.5	2
^{138}I $^{(89)}$Br $^{(93,94)}$Rb	9.1 6.3 ~ 9	3
^{139}I (Cs, Sb, Te) $^{(90,92)}$Br $^{(93)}$Kr	3 (2.3 --- 3.5) 2.3 ~ 2.2	4
(^{140}I, Kr)	0.7	5
(Br, Rb, As, …)	0.3	6

Source of data in table :
G.R Keepin, “Physics of Nuclear Kinetics” Addison-Wesley publishing
Company Inc.
(modified for present talk)

Delayed neutron (3/3)

Table 2 Six-DN-group precursor data of ^{235}U fission with thermal neutron

Group i	Mean-life τ_i [s]	Decay const. λ_i [s^{-1}]	DN fraction β_i
1	80.65	0.0124	0.000215
2	32.79	0.0305	0.001424
3	9.01	0.111	0.001274
4	3.32	0.301	0.002568
5	0.88	1.14	0.000748
6	0.33	3.01	0.000273
	Mean 13.04	Mean 0.0767	Sum β =0.0065

（A contribution from ^{87}Br is dominant in the 1st. group.）

Mean of six-group parameters :

$$\bar{\tau} = \frac{1}{\beta}\sum_{i=1}^{6}\beta_i \tau_i \qquad (2)$$

$$\bar{\lambda} = \frac{1}{\bar{\tau}} \qquad (3)$$

➡ **One-DN-group approximation**

Source of data in table : G.R Keepin, “Physics of Nuclear Kinetics” Addison-Wesley publishing Company Inc.,

Neutron multiplication with "mean" lifetime (1/3)

We consider the increase or decrease of total number of neutrons $N(t)$ at t in the reactor by using the neutron lifetime l.

$$\frac{dN(t)}{dt} = -\frac{N(t)}{l} + \frac{N(t)}{l} k_{eff} = \frac{k_{eff}-1}{l} N(t) \qquad (6)$$

Neutron number change rate

Neutron loss rate

Neutron production rate

A kind of the reactor kinetics equation

Production rate
= loss rate x k_{eff}

Similar to the decay of "radioactive nuclide"

$$\frac{d\,n(t)}{d\,t} = -\lambda\,n(t) = -\frac{n(t)}{\tau}$$

Nuclide number change rate

λ is the decay const.

$\tau = 1/\lambda$ is the life.

$$k_{eff} = \frac{\text{Production rate}}{\text{Loss rate}}$$

Neutron multiplication ... (2/3)

$$\frac{dN(t)}{dt} = \frac{k_{eff} - 1}{l} N(t) \quad (6)$$

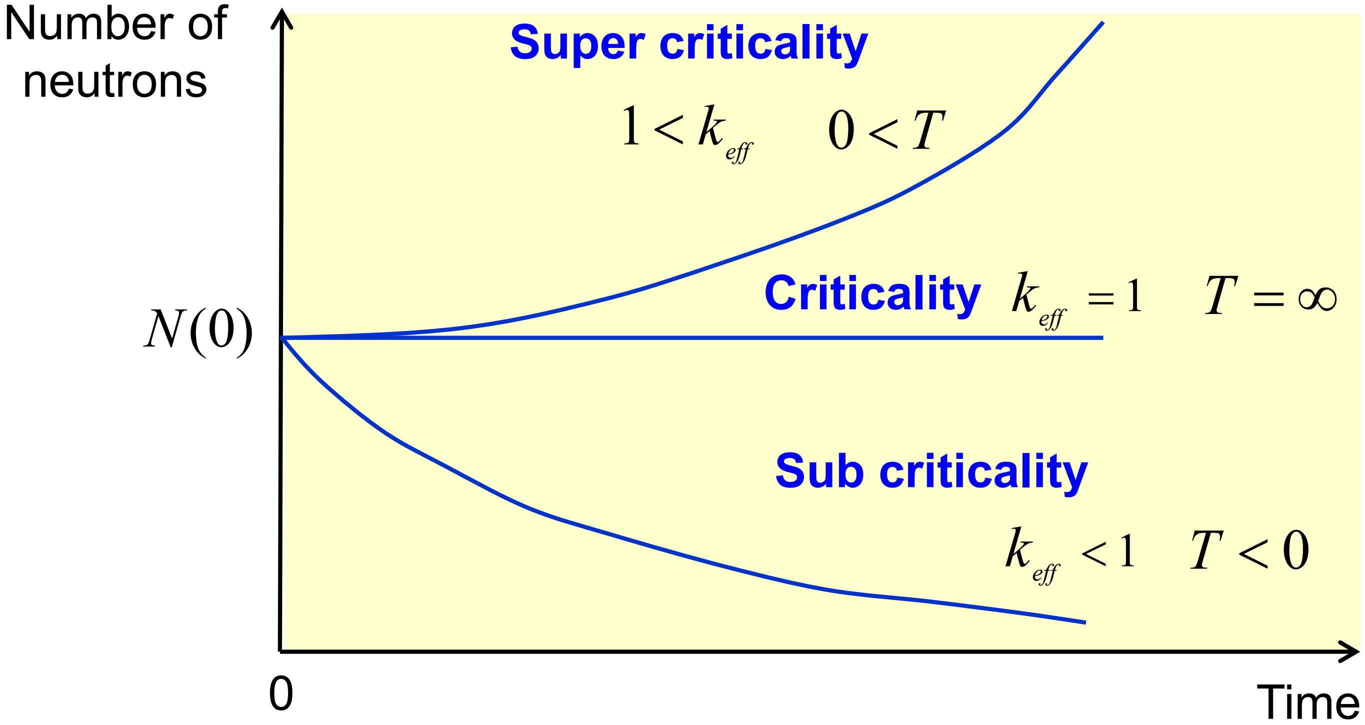
$$N(t) = N(0)\ \exp\left(\frac{k_{eff} - 1}{l} t\right) = N(0)\ \exp\left(\frac{t}{T}\right) \quad (7)$$

Initial number of neutrons

$$T \equiv \frac{l}{k_{eff} - 1} \quad (8)$$

T is called the **reactor period**.

In time T, the neutrons either increases or decreases by the factor e (~2.72).



Neutron multiplication ... (3/3)

If the DN does not exist, the (real) neutron lifetime l = 100 [μs] is used for the reactor period calculation :

$$T = \frac{l}{k_{eff} - 1} = \frac{1 \times 10^{-4}}{0.001} = 0.1 \quad \text{[s]}$$

- In one second, the reactor pass through 10 periods and power increases by a factor of $e^{10} = 2.2 \times 10^4$.
- When the reactor is initially operating at 1 [MW], the power increases to 22,000 [MW] in only one second, if no power feedback arises.
- A reactor with such a short period would be very difficult to control.

⇩

Demonstrates the importance of DN in the reactor kinetics and control.

- So far, we have considered the delay of DN emission roughly and “implicitly" by using the "pseudo-lifetime of DN l_d ".
- For more detailed analysis, we treat the decay of precursor "explicitly", in the following pages.

Derivation of reactor kinetics equations (1/6)

- The kinetics equation is rigorously derived from the time-dependent neutron diffusion equation. But we have to handle many equations and need much time in this process, so we take another easy route to derive the kinetics equations as follows.

  - Consider not only the number of neutrons $N(t)$, but also **fictitious number of precursors $C(t)$** so that one DN precursor emits one DN.

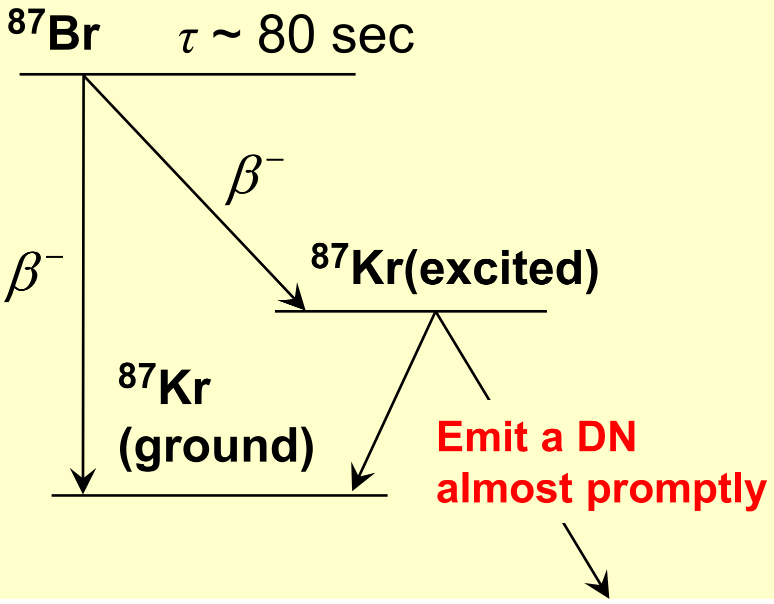
  - Introduce the **time delay of DN emission "explicitly"** to the previously derived neutron multiplication equation :

$$\frac{d\,N(t)}{d\,t} = -\frac{N(t)}{\bar{l}} + \frac{N(t)}{\bar{l}} k_{eff} \qquad (1)$$

Loss rate **Production rate**

Example of DN emission

Only a part of ^{87}Br decays results in DN emission.

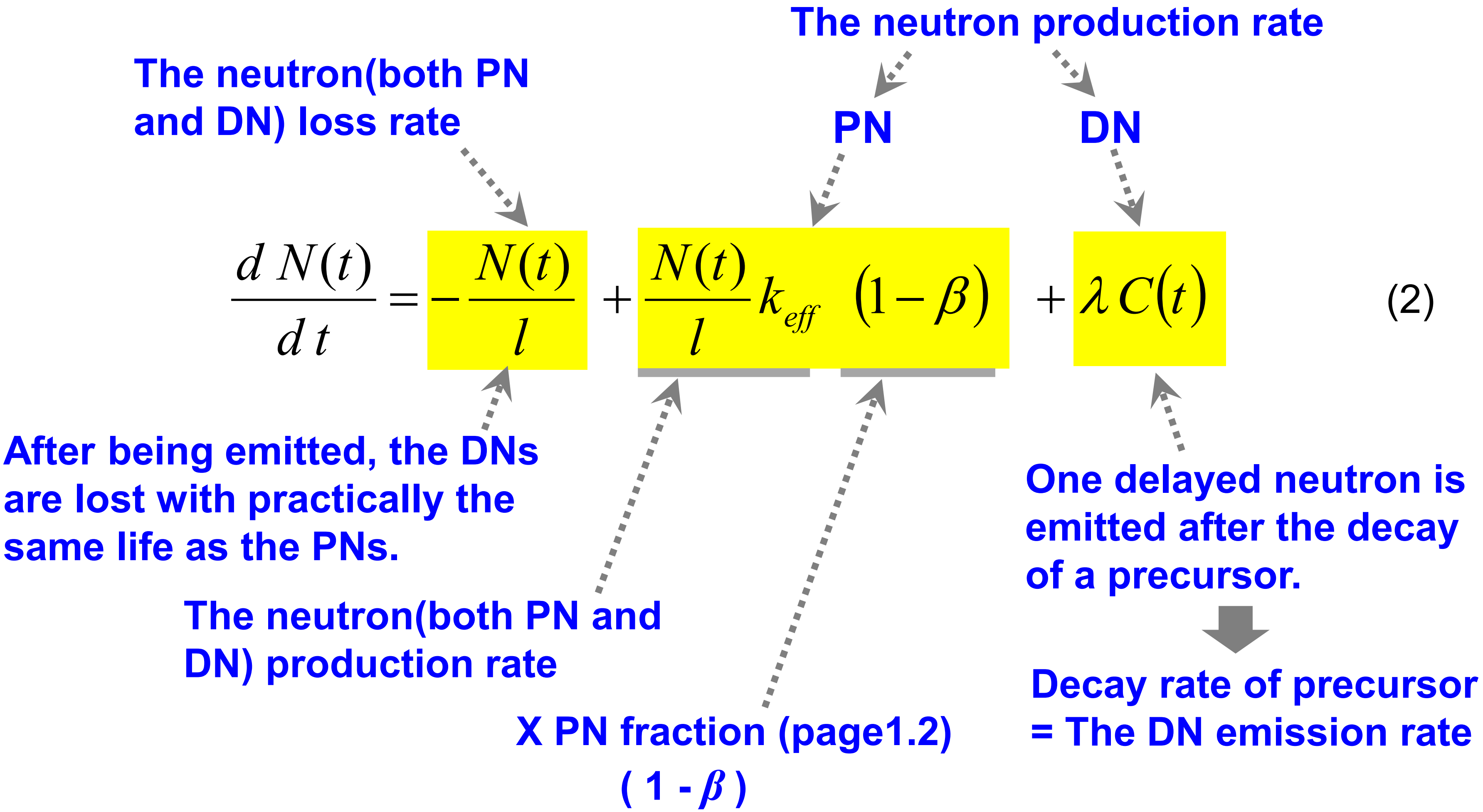


Derivation of ... (2/6)

We first use the one-DN-group approximation, and later, we move to the six-DN-group model.

We treat the delay of DN emission explicitly, so we use the "real" life time.

■ Neutron number change rate



The neutron production rate

The neutron(both PN and DN) loss rate

PN

DN

$$\frac{d\,N(t)}{d\,t} = -\frac{N(t)}{l} + \frac{N(t)}{l}k_{eff}\,(1-\beta) + \lambda\,C(t) \qquad (2)$$

After being emitted, the DNs are lost with practically the same life as the PNs.

One delayed neutron is emitted after the decay of a precursor.

The neutron(both PN and DN) production rate

Decay rate of precursor = The DN emission rate

X PN fraction (page1.2)
(1 - β)

Derivation of ... (3/6)

■ Precursor number change rate

One DN corresponds to one precursor.

↓

Production rate of precursor are assumed to be equivalent to the production rate of DN(before emission into the reactor).

$$\frac{d\,C(t)}{d\,t} = \overbrace{\underbrace{\frac{N(t)}{l}\,k_{eff}}_{\textbf{The neutron(both PN and DN) production rate}}\ \underbrace{\beta}_{\textbf{X The DN fraction (}\beta\textbf{)}}}^{\textbf{The precursor production rate}} - \overbrace{\lambda\,C(t)}^{\textbf{The loss rate}} \qquad (3)$$

Derivation of ... (4/6)

- **Coupled differential equations for neutrons and precursors.**

The PN and DN loss rate

The PN production rate

The DN production rate

Neutrons

$$\frac{d\,N(t)}{d\,t} = -\frac{N(t)}{l} + \frac{N(t)}{l} k_{eff} (1-\beta) + \lambda C(t) \qquad (4)$$

The DNs are not emitted promptly. ➡ **The DNs are emitted after the precursor decay.**

Precursors

$$\frac{d\,C(t)}{d\,t} = \frac{N(t)}{l} k_{eff}\, \beta - \lambda C(t) \qquad (5)$$

These are the **reactor kinetics equation**
(with one-DN-group approximation).

Derivation of ... (5/6)

We modify the equations in the former page.

$$\begin{cases} \dfrac{d\,N(t)}{d\,t} = -\dfrac{N(t)}{l} + \dfrac{N(t)}{l} k_{eff}(1-\beta) + \lambda\,C(t) \\ \qquad = \boxed{\dfrac{k_{eff}(1-\beta)-1}{l} N(t)} + \lambda\,C(t) \\ \qquad = \dfrac{1-\beta-1/k_{eff}}{l/k_{eff}} N(t) + \lambda\,C(t) \qquad (6) \\ \dfrac{d\,C(t)}{d\,t} = \dfrac{k_{eff}\,\beta}{l} N(t) - \lambda\,C(t) = \dfrac{\beta}{l/k_{eff}} N(t) - \lambda\,C(t) \qquad (7) \end{cases}$$

Production rate of PNs

― loss rate of (PN+DN)s

We use the reactivity $\rho \equiv \dfrac{k_{eff}-1}{k_{eff}} = 1 - 1/k_{eff}$ and introduce "neutron generation time" $\Lambda \equiv l/k_{eff}$.

Λ is a mean time between the birth of a neutron(A) and the subsequent production of another neutron due to the fission of neutron(A) in the fuel. At the criticality, the loss rate of neutrons equals to the production rate, so the Λ and l takes the same value. Near the criticality, the values of both parameters are practically the same.

Derivation of ... (6/6)

We arrive an alternative form of the kinetics equations.

$$\begin{cases} \dfrac{d\,N(t)}{d\,t} = \dfrac{\rho-\beta}{\Lambda}N(t) + \lambda\,C(t) & (8) \\[2ex] \dfrac{d\,C(t)}{d\,t} = \dfrac{\beta}{\Lambda}N(t) - \lambda\,C(t) & (9) \end{cases}$$

Production rate of PNs
— loss rate of (PN+DN)s

With the six-DN-group model, each precursor group needs its equation. We then have the kinetics equations(7 coupled differential equations) of

$$\begin{cases} \dfrac{d\,N(t)}{d\,t} = \dfrac{\rho-\beta}{\Lambda}N(t) + \sum_{i=1,6}\lambda_i\,C_i(t) & (10) \\[2ex] \dfrac{d\,C_i(t)}{d\,t} = \dfrac{\beta_i}{\Lambda}N(t) - \lambda_i\,C_i(t) \quad (i=1,6) & (11) \end{cases}$$

Sum of six group precursors

$$\left(\beta = \sum_{i=1,6}\beta_i\right)$$

Characteristics of kinetics equations

$$\frac{d\,N(t)}{d\,t} = \frac{\rho-\beta}{\Lambda} N(t) + \lambda\, C(t)$$

Contribution of DNs for the neutron increase

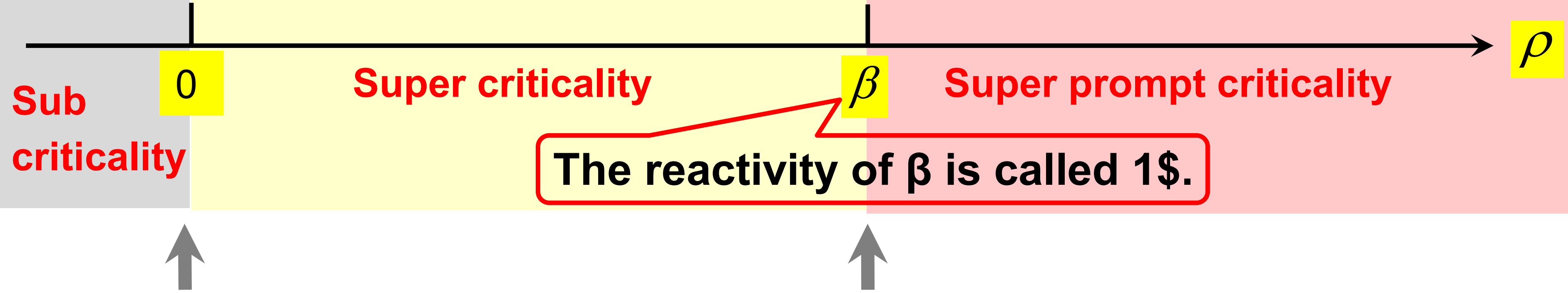
Production rate of PNs − loss rate of (PN+DN)s

$$\frac{\rho-\beta}{\Lambda} N(t) < 0$$

- Super criticality with both PNs and DNs
- Sub criticality with solely PNs
- Needs contribution of DNs to increase the neutron.

$$0 < \frac{\rho-\beta}{\Lambda} N(t)$$

- Super criticality with only PNs
- Contribution of DN is not necessary for the neutron increase.
- Very short reactor period



Sub criticality

Super criticality

Super prompt criticality

The reactivity of β is called 1$.

Criticality with both PNs and DNs

= delayed criticality

Criticality solely with PNs

= prompt criticality

Summary of Chap.1

■ Summary

- ✓ We derived the reactor kinetics equation and found the followings.
- ✓ Under the prompt criticality, the contribution of DNs is needed to increase the reactor power.
- ✓ Over the prompt criticality, the PNs can increase the rector power with very short reactor period.
- ✓ The prompt criticality must be avoided in usual reactor operation.

2. Reactor response for a simple reactivity change

Step change of reactivity

- As a typical and simple case, we consider the time-dependent reactor behavior, after sudden insertion of small reactivity of far below 1 dollar to the steady state critical reactor.
- This reactivity insertion is usually called "step change of reactivity".
- For simplicity, we do not consider reactivity feedback effects due to reactor power and temperature change.
- Although it's a simple model of reactivity change, we can learn a typical reactor response with this reactivity change.

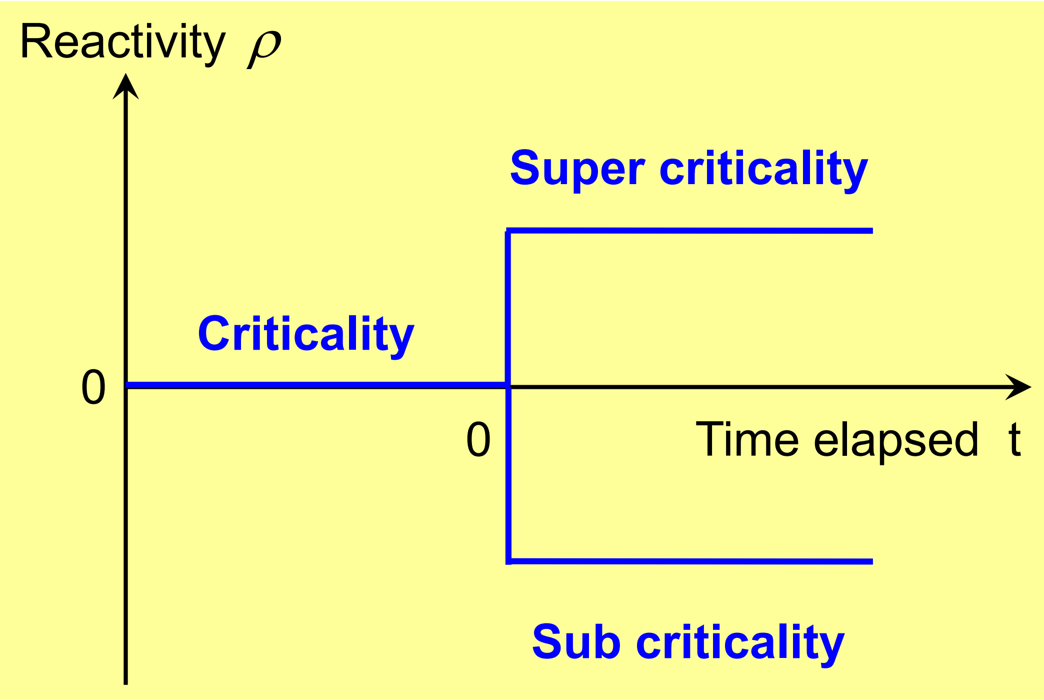


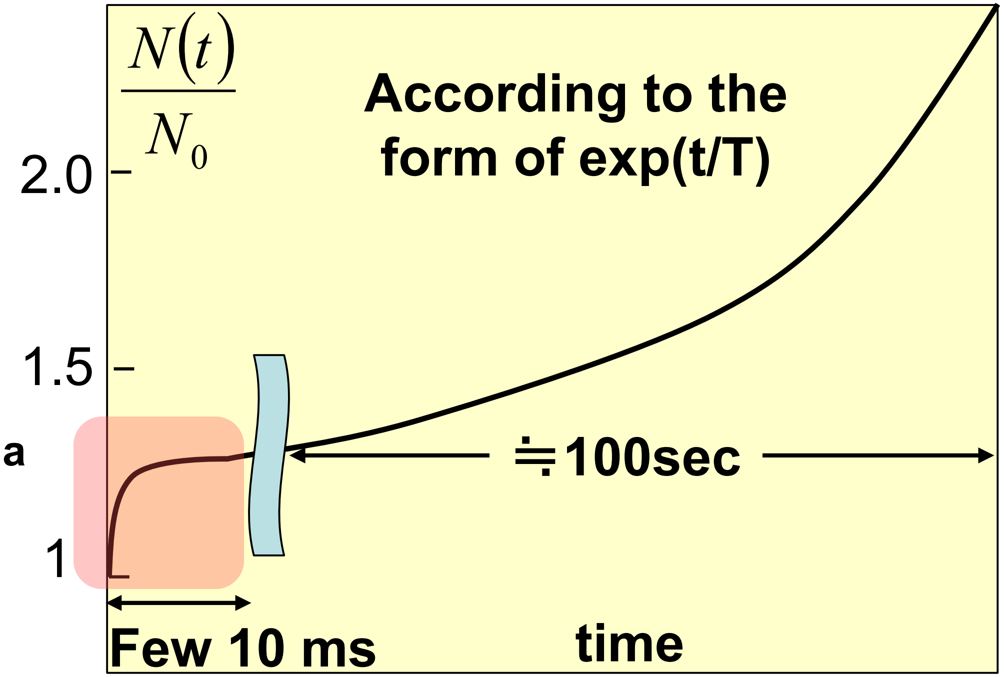
Fig.1 Step change of reactivity at t=0

Final solution -before solving-

- **After several approximation, a solution consisting of two exponential terms, stable and transient period, is obtained.**

$$\frac{N(t)}{N_0} \approx \frac{\beta_{eff}}{\beta_{eff}-\rho}\boxed{\exp\left(\frac{\lambda\,\rho}{\beta_{eff}-\rho}t\right)} - \frac{\rho}{\beta_{eff}-\rho}\boxed{\exp\left(-\frac{\beta_{eff}-\rho}{\Lambda}t\right)}$$

- **Transient changes that occur and disappear within a few 10 ms after reactivity is initiated.**



- **After transient change, stable change that has 100-sec occurs.**

Based on final solution, focusing on each of the transient and stable changes, we proceed calculation.

Solution of kinetics equations with one-DN-group approximation (1/8)

As a first step, we use the one-DN-group approximation. An analytical solution which illustrates the reactor response can be easily obtained with this approximation, though it is less accurate than the six-DN-group model.

Table 2 Six-DN-group precursor data of ^{235}U fission with thermal neutron

Group i	Mean-life τ_i [s]	Decay const. λ_i [s^{-1}]	DN fraction β_i
1	80.65	0.0124	0.000215
2	32.79	0.0305	0.001424
3	9.01	0.111	0.001274
4	3.32	0.301	0.002568
5	0.88	1.14	0.000748
6	0.33	3.01	0.000273
	Mean 13.04	Mean 0.0767	Sum β =0.0065

$$\tau \equiv \frac{1}{\beta}\sum_{i=1}^{6}\beta_i\,\tau_i \quad \Rightarrow \quad \tau = \frac{1}{\lambda} \quad \Rightarrow \quad \frac{1}{\lambda} \equiv \frac{1}{\beta}\sum_{i=1,6}\frac{\beta_i}{\lambda_i}$$

One-DN-group approximation uses these mean values.

Solution of ... with one-DN-group approximation (2/8)

■ Solution at the critical steady-state

The kinetics equations with one-DN-group approximation are

$$\begin{cases} \dfrac{d\,N(t)}{d\,t} = \dfrac{\rho - \beta}{\Lambda} N(t) + \lambda\, C(t) & (1) \\[2ex] \dfrac{d\,C(t)}{d\,t} = \dfrac{\beta}{\Lambda} N(t) - \lambda\, C(t). & (2) \end{cases}$$

Suffix 0 denotes the value at the critical steady-state

At the critical steady-state,

the ρ = 0, dN/dt = 0 and dC/dt = 0 , ➡

$$\frac{\beta}{\Lambda} N_0 - \lambda\, C_0 = 0\,. \qquad (3)$$

For Λ=10 (μ sec)

$$C_0 = \frac{\beta}{\lambda\,\Lambda} N_0 = \frac{0.0065}{0.077 \times 0.00001} N_0 \approx 10000 \times N_0. \qquad (4)$$

The population of DN precursors is, typically, about 10000 times greater than the neutron population in a critical reactor.

The accumulated DN precursors play a role of neutron source which controls the reactor kinetics under ' normal operating conditions '.

Solution of ... with one-DN-group approximation (3/8)

To solve the Kinetics equations, an exponential form for the $N(t)$ and $C(t)$ are assumed, because their derivatives with respect to time are proportional to themselves :

$$N(t) = A_N \exp(\omega t) \qquad C(t) = A_C \exp(\omega t) \qquad (5)$$

Introducing these expressions to the kinetics equations, we have

$$\left\{ \begin{aligned} \omega A_N \cancel{e^{\omega t}} &= \frac{\rho - \beta}{\Lambda} A_N \cancel{e^{\omega t}} + \lambda A_C \cancel{e^{\omega t}} \quad \Longrightarrow \quad \omega A_N = \frac{\rho - \beta}{\Lambda} A_N + \lambda A_C \qquad (6) \\ \omega A_C \cancel{e^{\omega t}} &= \frac{\beta}{\Lambda} A_N \cancel{e^{\omega t}} - \lambda A_C \cancel{e^{\omega t}} \quad \Longrightarrow \quad A_C = \frac{1}{\omega + \lambda} \frac{\beta}{\Lambda} A_N \qquad (7) \end{aligned} \right.$$

Solution of ... with one-DN-group approximation (4/8)

By substituting Eq.(7) into Eq.(6) and arranging :

$$\left\{\frac{\rho-\beta}{\Lambda}-\omega+\frac{\lambda}{\omega+\lambda}\frac{\beta}{\Lambda}\right\}A_N=0\,. \qquad (8)$$

The A_N should not always be 0, therefore

$$\frac{\rho-\beta}{\Lambda}-\omega+\frac{\lambda}{\omega+\lambda}\frac{\beta}{\Lambda}=0\,. \qquad (9)$$

$$\omega^2+\left(\frac{\beta-\rho}{\Lambda}+\lambda\right)\omega-\frac{\lambda\,\rho}{\Lambda}=0 \qquad (10)$$

"Characteristic equation" for the coupled differential equation

Two roots of Eq.(10) are given with some approximations(see appendix for details) by

$$\omega_+\approx\frac{\lambda\,\rho}{\beta-\rho}\,, \qquad \omega_-\approx-\frac{\beta-\rho}{\Lambda}\,. \qquad (11)$$

Solution of ... with one-DN-group approximation (5/8)

Furthermore, Eq.(9) can be modified as follows :

$$\rho = \omega \Lambda + \frac{\omega}{\omega + \lambda} \beta \qquad (12)$$

$$\left(\Lambda = l / k_{eff} \right)$$

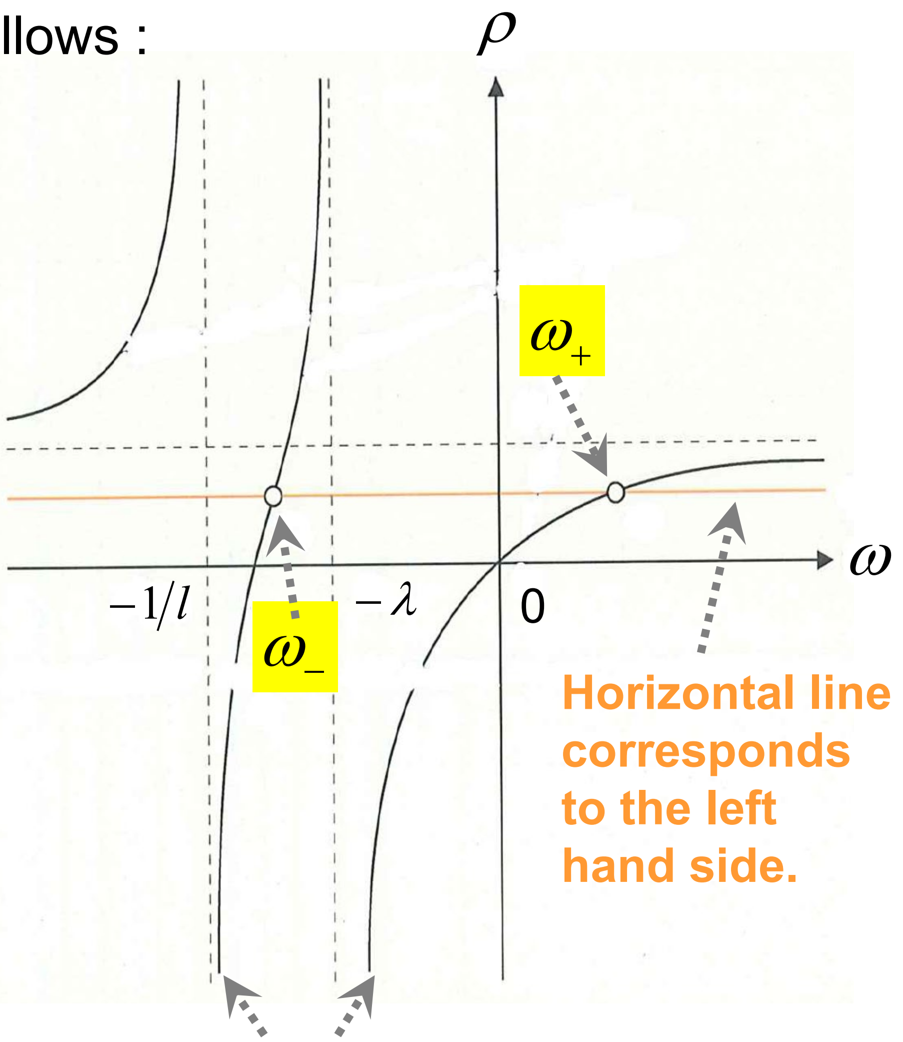
$$\rho = \omega \frac{l}{k_{eff}} + \frac{\omega}{\omega + \lambda} \beta \qquad (13)$$

$$\left(1 / k_{eff} = 1 - \rho \right)$$

$$\rho = \omega l \left(1 - \rho \right) + \frac{\omega}{\omega + \lambda} \beta \qquad (14)$$

We finally arrive "**Inhour equation**" (one-DN-group in this case) :

$$\rho = \frac{\omega l}{1 + \omega l} + \frac{1}{1 + \omega l} \frac{\omega}{\omega + \lambda} \beta . \qquad (15)$$



Horizontal line corresponds to the left hand side.

Curves are trace of the right hand side.

Fig.2 Graphical solution of inhour equation

In the early days of reactor technology, values of ω were quoted in "inverse hours" The values of ρ such that ω=1 hr^{-1} is "one inhour". Source : A. F. Henry, "Nuclear-Reactor Analysis" The MIT press

Solution of ... with one-DN-group approximation (6/8)

The characteristic equation has two roots ω_+ and ω_- .

The general solution for $N(t)$ is therefore given by

$$N(t) = A\exp(\omega_+ t) + B\exp(\omega_- t), \qquad (16)$$

with arbitrary constants A and B, which are determined from initial conditions.

The derivative of $N(t)$ with respect to time :

$$\frac{d\,N(t)}{d\,t} = A\omega_+ \exp(\omega_+ t) + B\omega_- \exp(\omega_- t). \qquad (17)$$

$$\left.\frac{d\,N(t)}{d\,t}\right|_{t=0} = A\omega_+ + B\omega_- . \qquad (18)$$

The general solution for $C(t)$ is given by a similar formula of

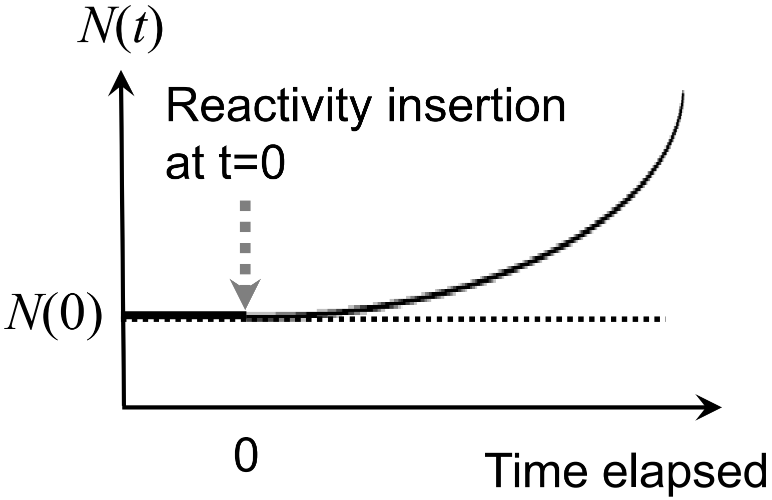
$C(t) = A'\exp(\omega_+ t) + B'\exp(\omega_- t),$ with arbitrary constants A' and B' .

Solution of ... with one-DN-group approximation (7/8)

The initial conditions

Before reactivity insertion, $N(t)$ and $C(t)$ take constant values $N(0)$ and $C(0)$

$$N(0) = A + B \, , \qquad (19)$$



$$\left.\frac{d\,C(t)}{d\,t}\right|_{t<0} = \frac{\beta}{\Lambda} N(0) - \lambda\, C(0) = 0$$

$$\Rightarrow \quad \frac{\beta}{\Lambda} N(0) = \lambda\, C(0). \qquad (20)$$

Just after reactivity insertion, $N(t)$ and $C(t)$ stays $N(0)$ and $C(0)$ in a very short period :

$$\left.\frac{d\,N(t)}{d\,t}\right|_{t=+0} = \frac{\rho-\beta}{\Lambda} N(0) + \lambda\, C(0) = \frac{\rho}{\Lambda} N(0).$$

$\left.\frac{d\,N(t)}{d\,t}\right|_{t=0}$ is also given by Eq.(18). $\Rightarrow$ $$A\,\omega_+ + B\,\omega_- = \frac{\rho}{\Lambda} N(0) \qquad (21)$$

Solution of ... with one-DN-group approximation (8/8)

A and B are determined from Eqs.(18) and (21) :

$$\begin{cases} A = \dfrac{\dfrac{\rho}{\Lambda}-\omega_-}{\omega_+-\omega_-}N(0) = -\dfrac{(\beta/\Lambda)N(0)}{\left(\dfrac{\rho-\beta}{\Lambda}\right)\left(\cancel{\dfrac{\lambda\,\rho\,\Lambda}{(\rho-\beta)^2}}+1\right)} \approx \dfrac{\beta}{\beta-\rho}N(0) & (22) \\ B \approx -\dfrac{\rho}{\beta-\rho}N(0) & (23) \end{cases}$$

$$\lambda\,\rho\,\Lambda << (\rho-\beta)^2$$

- **The final solution becomes**

$$N(t) \approx \frac{\beta}{\beta-\rho}N(0)\exp\left(\underbrace{\frac{\lambda\,\rho}{\beta-\rho}t}_{(\omega_+ t)}\right) - \frac{\rho}{\beta-\rho}N(0)\exp\left(\underbrace{-\frac{\beta-\rho}{\Lambda}t}_{(\omega_- t)}\right). \qquad (24)$$

- By introducing the contribution of delayed neutrons to the multiplication, two exponential terms (two periods $1/\omega_+$ and $1/\omega_-$) appeared.
- This solution has good accuracy only when the inserted reactivity is small, because several approximation are used to derive this.

Reactor response after positive reactivity insertion (1/3)

■ Example

Neutron generation time $\Lambda \sim l$ = 100 [μs] (1×10^{-4} [s])

DN fraction β = 0.0065   DN precursor mean decay const. λ = 0.077 [s^{-1}]

Reactivity insertion of +0.1 [%Δk/k] (about 0.15 [dollar])

Neutron number normalized by it's initial value

$$\frac{N(t)}{N(0)} \approx \frac{\beta}{\beta-\rho}\exp\left(\frac{\lambda\,\rho}{\beta-\rho}t\right) - \frac{\rho}{\beta-\rho}\exp\left(-\frac{\beta-\rho}{\Lambda}t\right)$$

$$= \boxed{1.18}\exp\left(\frac{1}{\boxed{69}}t\right)\boxed{-0.18}\exp\left(\frac{1}{\boxed{-0.018}}t\right) \tag{1}$$

Calculate amplitudes and periods.

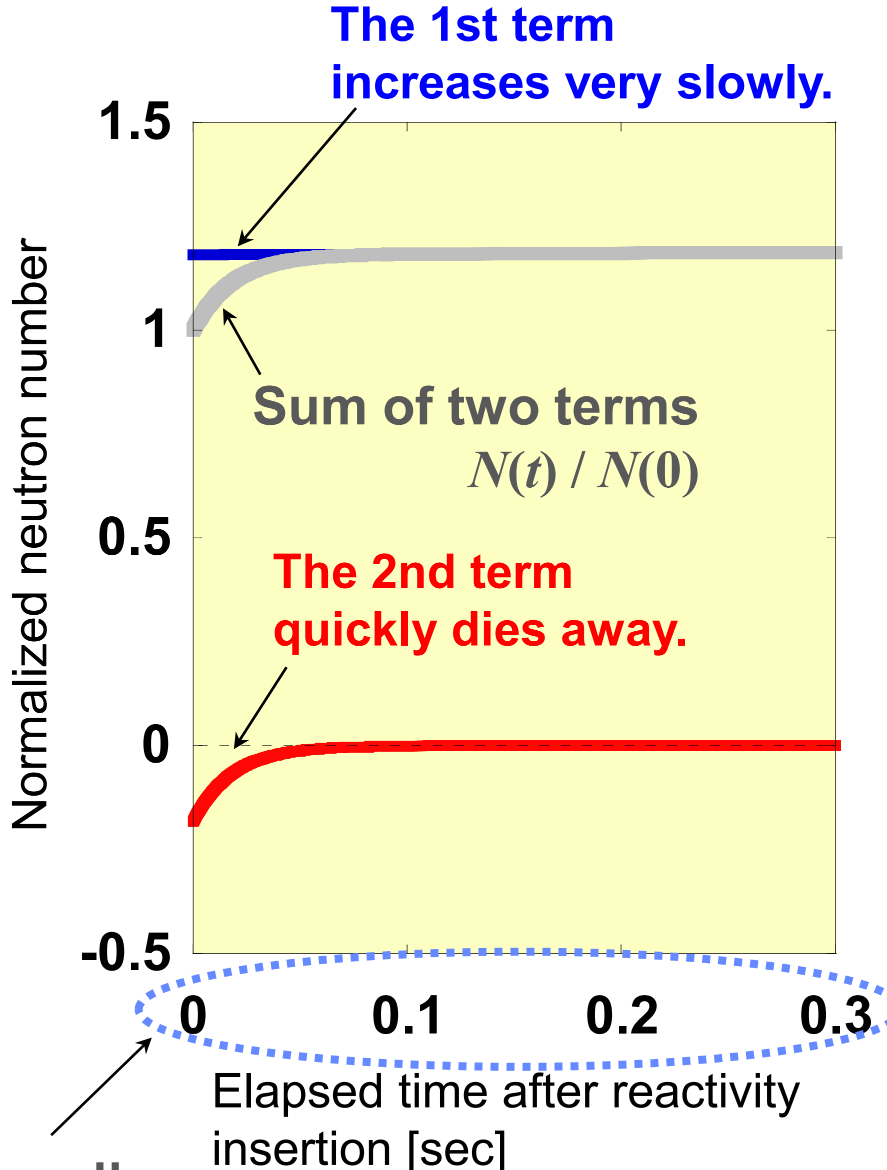
Reactor response after positive reactivity insertion (2/3)

Response **just after** the reactivity insertion

- The 1st term practically does not change in this short time range because of its long period.
- The 2nd term quickly dies away with the negative and short period

- The $N(t)$ quickly increases.
- This "rapid change" is caused by PNs and called the **prompt jump**.
- But the reactor is sub criticality with solely the PNs, so this "rapid increase" saturates at a level :

$$N_{jmp} = \frac{\beta}{\beta - \rho} N(0). \qquad (2)$$

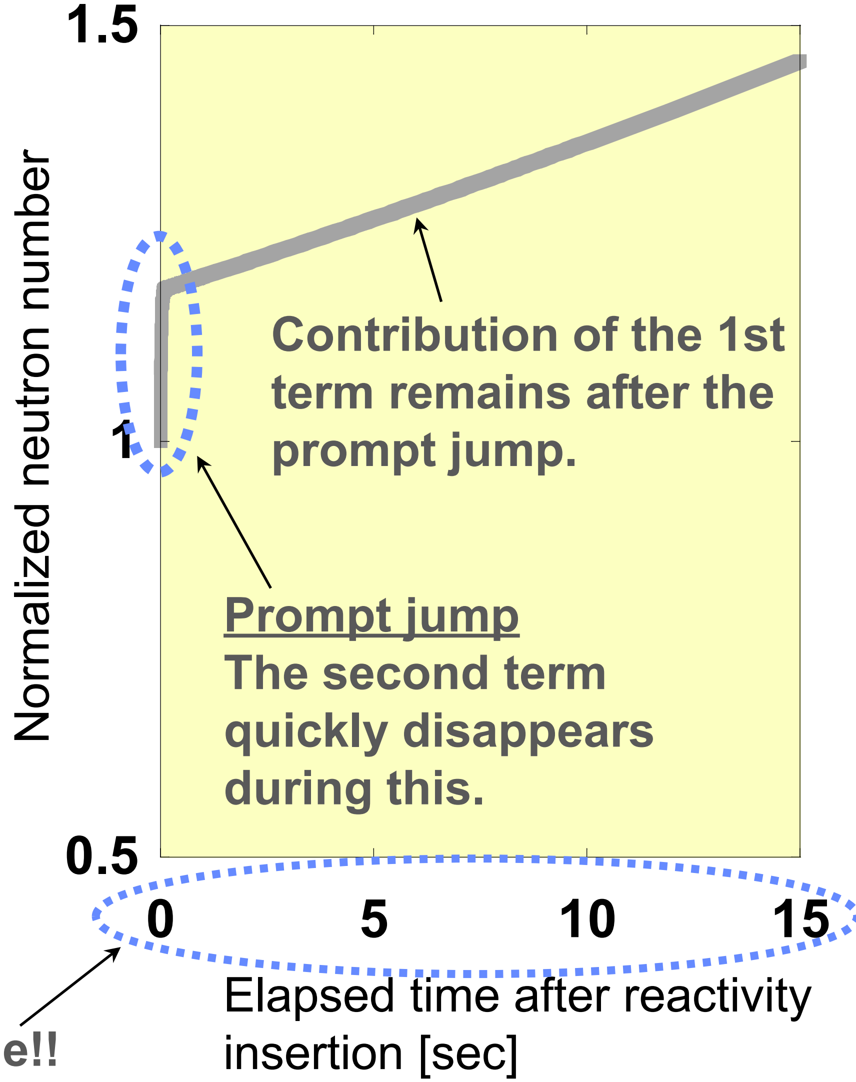


Very short time range!!

Reactor response after positive reactivity insertion (3/3)

Response after the prompt jump

- The second term has already been disappeared, that means the rapid increase of PNs has been saturated.
- Only the 1st term having long period remains.
- The $N(t)$ increases slowly.
- This slow increase is dominated by that of DNs emitted from the decay of precursors.



Contribution of the 1st term remains after the prompt jump.

Prompt jump
The second term quickly disappears during this.

Long time range!!

Reactor response after negative reactivity insertion

Example

ρ = - 0.1 %Δk/k
(-0.15 $)

$$\frac{N(t)}{N(0)} = 0.87\exp\left(\frac{t}{-97}\right) + 0.13\exp\left(\frac{t}{-0.013}\right) \tag{3}$$

Negative and long period

Negative and very short period

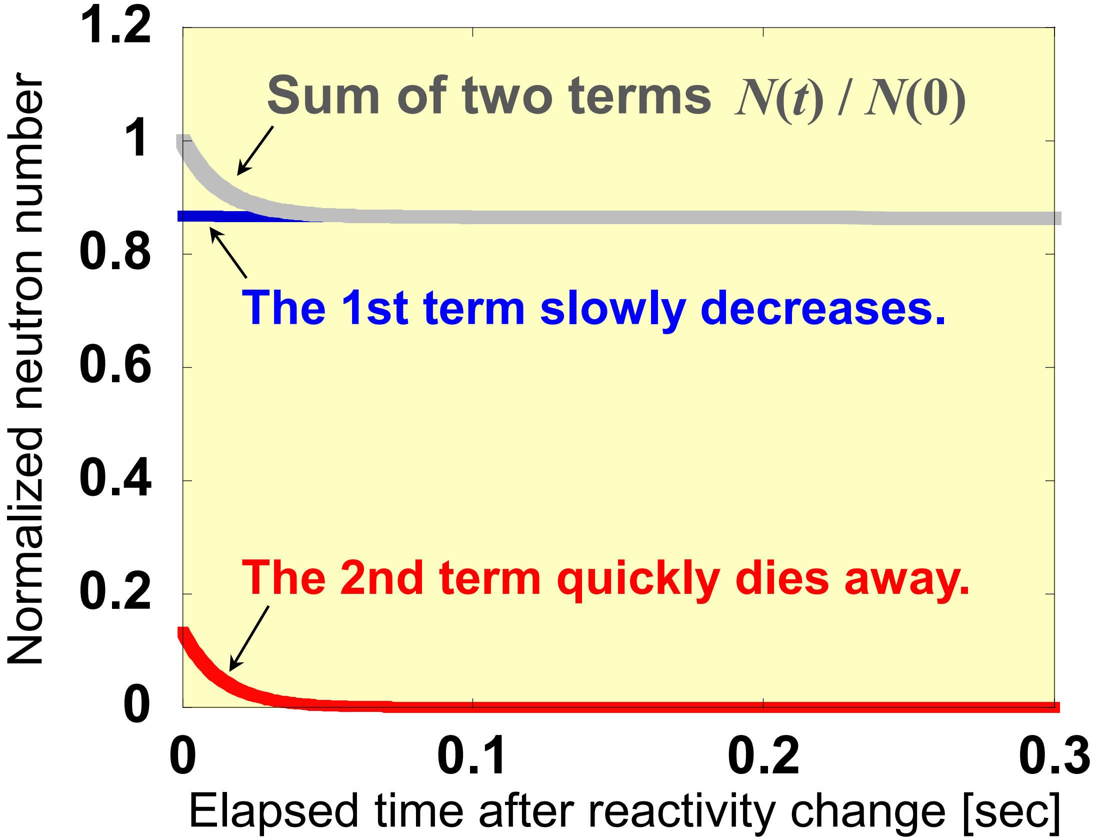
Response

- The $N(t)$ quickly decreases to

$$N_{jmp} = \frac{\beta}{\beta - \rho} N(0).$$

due to the prompt jump.

- After the jump, the $N(t)$ slowly decreases.



Delayed neutron production rate constant approximation

If it is approximated that the number of precursor does not change and remains at C_0 for a shorter time than the lifetime of the precursor, such as during and immediately after the prompt jump after the reactivity is applied,

$$\frac{d\,n(t)}{d\,t}=\frac{\rho_0-\beta}{\Lambda}n(t)+\lambda C(t)\approx\frac{\rho_0-\beta}{\Lambda}n(t)+\lambda C_0 \qquad \lambda C_0=(\beta/\Lambda)n_0$$

2.4 (3)

$$=\frac{\rho_0-\beta}{\Lambda}n(t)+\frac{\beta}{\Lambda}n_0$$

This types of linear differential equation can be solved,

$$\frac{n(t)}{n(0)}=\frac{\beta}{\beta-\rho_0}-\frac{\rho_0}{\beta-\rho_0}\exp\left(\frac{\rho_0-\beta}{\Lambda}t\right)$$

Term of transient period

After the reactivity is applied, the transient periodic term decays to negligible in a short time and the prompt jump is terminated. The ratio of neutron densities before and after the jump is

$$\Rightarrow \quad \frac{n_1}{n_0}\approx\frac{\beta}{\beta-\rho_0}$$

Prompt jump approximation (1/2)

✓ General definition of reactor period.

Typically, the reactor period is defined as the reciprocal of the relative rate of change in neutron number (or reactor power) over time

$$T(t)=\frac{1}{(1/N(t))\,d\,N(t)/d\,t}\quad\left(=\frac{N(t)}{d\,N(t)/d\,t}\right)\quad=\frac{1}{d\ln N(t)/d\,t}$$

Comparison with transient periods as stable periods after immediate jumps.

$$\left|\frac{\Lambda}{\rho-\beta}\right| << \left|\frac{N(t)}{d\,N(t)/d\,t}\right| \quad\Rightarrow\quad \left|\frac{d\,N(t)}{d\,t}\right| << \left|\frac{\rho-\beta}{\Lambda}N(t)\right|$$

(1 group approximation) transient period

stable period

After an immediate jump
Percentage change in $N(t)$

In the one-group approximation.

$$\left(\frac{\beta-\rho}{\lambda\,\rho}\right)$$

The neutron number change after the immediate jump is dominated by delayed neutrons and the speed is slow.

Prompt jump approximation (2/2)

✓ Prompt jump approximation

Using an approximation that neglects $dN(t)/dt$, except during the prompt jump.

$$\begin{cases} \cancel{\dfrac{dN(t)}{dt}} = \dfrac{\rho-\beta}{\Lambda}N(t)+\lambda C(t) \\[2ex] \dfrac{dC(t)}{dt} = \dfrac{\beta}{\Lambda}N(t)-\lambda C(t) \end{cases}$$

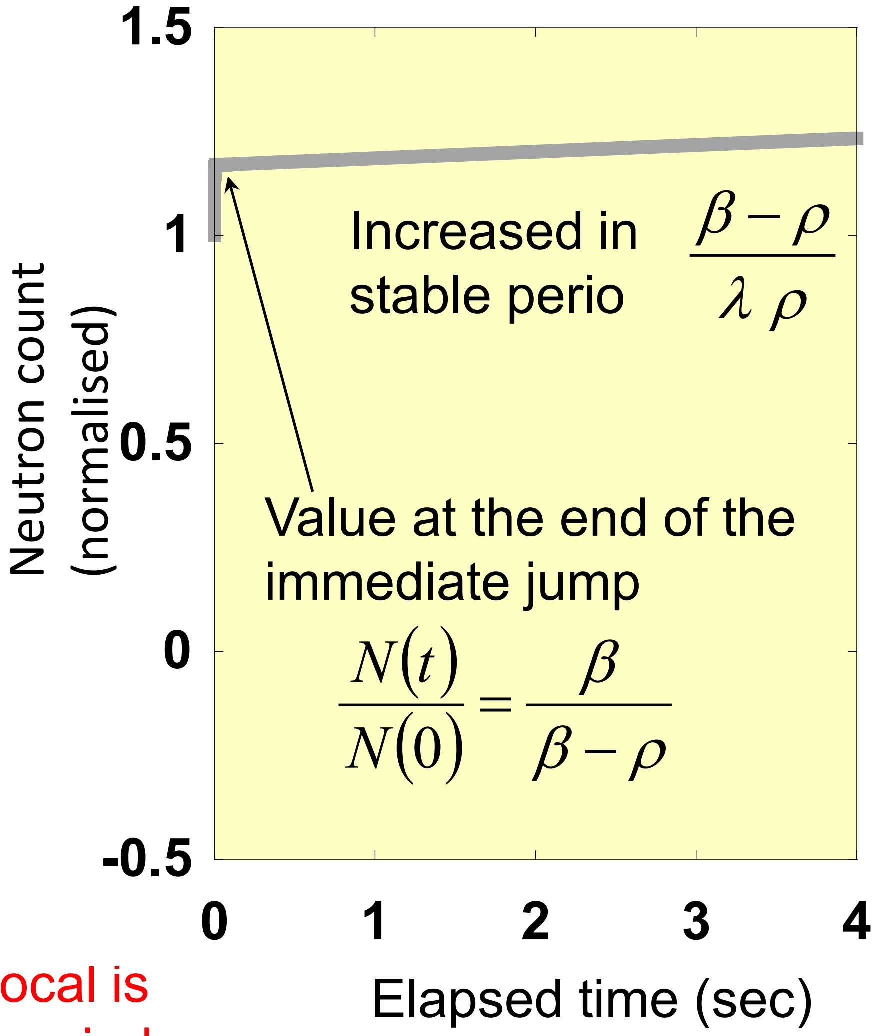
$C(t)$ elimination

$$\frac{dN(t)}{dt} = \frac{\lambda\rho}{\beta-\rho}N(t)$$

$$N(t) = \underline{\frac{\beta}{\beta-\rho}}N(0)\exp\left(\underline{\frac{\lambda\rho}{\beta-\rho}}t\right)$$

Default value is the value at the end of the prompt jump.

The reciprocal is the stable period.



Increased in stable perio $\dfrac{\beta-\rho}{\lambda\rho}$

Value at the end of the immediate jump

$$\frac{N(t)}{N(0)} = \frac{\beta}{\beta-\rho}$$

Reactor response Summary

Solution of kinetics equations with one-DN-group approximation :

$$N(t) \approx \frac{\beta}{\beta-\rho} N(0) \exp\left(\underbrace{\frac{\lambda\rho}{\beta-\rho} t}_{(\omega_+ t)} \right) - \frac{\rho}{\beta-\rho} N(0) \exp\left(\underbrace{-\frac{\beta-\rho}{\Lambda} t}_{(\omega_- t)} \right). \qquad (1)$$

	stable period (asymptotic period)	transient period
	$1/\omega_+ = (\beta-\rho)/(\lambda\,\rho)$	$1/\omega_- = -\Lambda/(\beta-\rho)$
$0 < \rho < \beta$	**Long and positive**	**Very short and negative**
$\rho < 0$	**Long and negative**	**Very short and negative**

- The prompt jump is caused by the rapid change of PNs number just after the reactivity insertion.
- This rapid change is "mathematically" shown by the die away of "transient period terms" in the solution of kinetics equations.

Summary of Chap.2

■ Summary

- ✓ We saw typical response of the reactor by solving analytically the reactor kinetics equation after step change of reactivity which is the most simple model of reactivity insertion.

■ Furthermore

- ✓ For more complicated reactivity change, it can be shown the reactor power changes with the stable period after the reactivity became constant and the transient terms disappeared.

Thank you for your kind attention on Nuclear Reactor kinetics

Appendix 1

Delayed neutron (4/4)

One-DN-group data

Table 3 DN precursor data of principal fissionable nuclides in one-DN-group approximation

Nuclide	Mean-life τ [s]	Decay const. λ [s^{-1}]	DN fraction β	Notice
^{235}U	13.04	0.0767	0.0065	Fission with thermal neutron
^{238}U	7.71	0.130	0.0148	Fission with fast neutron
^{239}Pu	14.76	0.0678	0.0021	Fission with thermal neutron

For example, low enriched uranium fuel used in PWR and BWR contains both ^{235}U and ^{238}U. The β in these reactor is calculated from β of these nuclides by appropriate averaging.

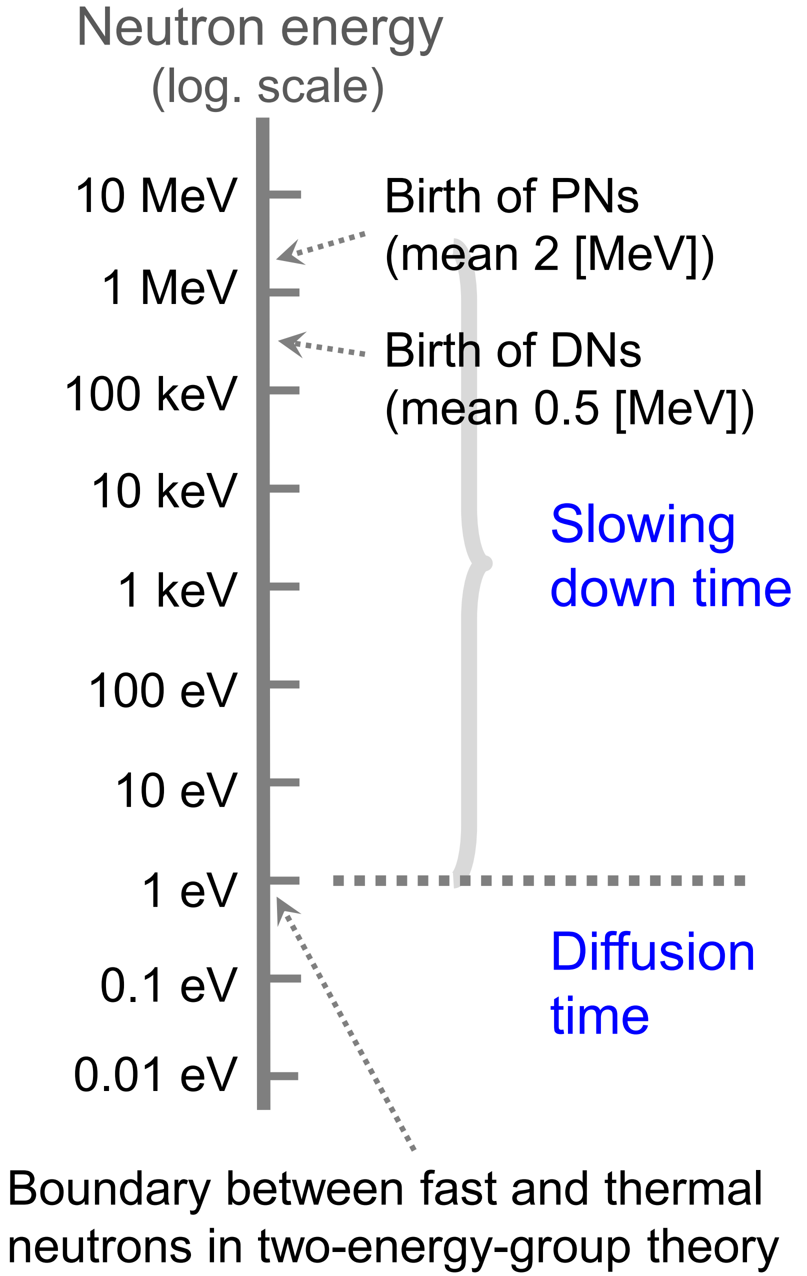
Neutron lifetime (1/2)

- The neutron lifetime l is the mean time from the birth of a neutron to the loss of this neutron in the reactor.
- **For the thermal reactor**, most of the neurons generated by the fission are lost by absorption or leakage after being slowed down(to the thermal energy) and diffusion as the thermal neutron.

Life time = ~~Slowing down time~~
+ Diffusion time
~ Diffusion time

(Slowing down time << Diffusion time)

- Typical neutron lifetime in the LWR (PWR and BWR) is 10 ~ 50 [μs]
- There is practically no difference of life time between PN and DN.

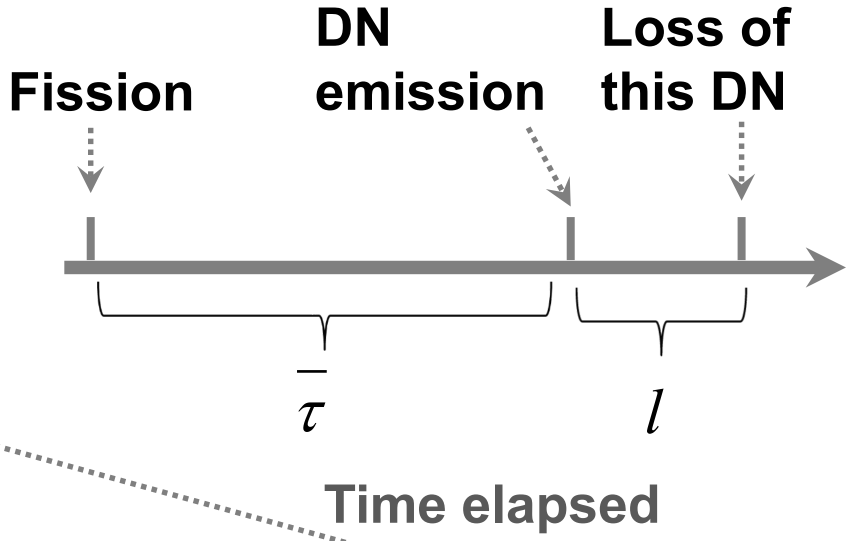


Boundary between fast and thermal neutrons in two-energy-group theory

Neutron lifetime (2/2)

- **For the DN**, we add the life of precursor to the neutron lifetime l to take account of the delay before it's emission due to precursor decay.

$$l_d \equiv l + \bar{\tau} \quad \left(\approx \bar{\tau}\right) \qquad (4)$$



$\left(l << \bar{\tau}\right)$

- We thus defined the **"pseudo"-lifetime l_d that includes the delay before DN emission**.

- We then calculate the "mean" of "**real" lifetime l of PN** and **"pseudo"-lifetime l_d of DN**.

$$\bar{l} = \underline{(1-\beta)}\, l + \underline{\beta}\, l_d \quad = (1-\beta)\, l + \beta \left(l + \bar{\tau}\right) \quad = l + \beta\, \bar{\tau} \quad \left(\approx \beta\, \bar{\tau}\right) \qquad (5)$$

weight for average

$l \sim 10^{-5}$ (s)

$\beta\, \tau \sim 10^{-2}\, 10 = 10^{-1}$ (s)

Neutron multiplication ... pseudo-lifetime

Example

Thermal neutron reactor fueled with ^{235}U , operating in the critical state

Neutron lifetime l = 100 [μs] (1 × 10^{-4} [s])

DN fraction β = 0.0065 **DN precursor mean-life τ = 13 [s]**

⬇

Sudden **reactivity insertion of +0.1 [%Δk/k]** (k_{eff} change 1.000 ➡1.001)

Calculate Mean of PN (real) lifetime l and DN "pseudo"-lifetime l_d by Eq.(5). :

$$\bar{l} = l + \beta\,\bar{\tau} = 1\times 10^{-4} + 0.0065\times 13 \approx 8.5\times 10^{-2} \quad \text{[s]}$$

Calculate the reactor period by Eq.(8). :

$$T = \frac{\bar{l}}{k_{eff}-1} = \frac{8.5\times 10^{-2}}{0.001} \approx 85 \quad \text{[s]}$$

Characteristics of kinetics equations (2/2)

Example of super prompt criticality

l =100 [μs] =1 × 10^{-4} [sec]

β =0.65 %

} Same as those in the example in p.1.11

ρ =0.75 [%Δk/k] ➡ super prompt criticality by 0.1 [%Δk/k])

$$\frac{d\,N(t)}{d\,t} = \frac{\rho-\beta}{\Lambda} N(t) + \cancel{\sum_{i=1,6} \lambda_i C_i(t)}$$

The contribution of DNs is small and neglected because the PNs dominate the neutron multiplication in this case.

$$\Downarrow$$

$$\frac{d\,N(t)}{d\,t} \approx \frac{\rho-\beta}{\Lambda} N(t)$$

$$\Downarrow$$

$$N(t) = N(0)\exp(t/T) \qquad T = \frac{\Lambda}{\rho-\beta} = \frac{1\times10^{-4}/1.0075}{0.75\times10^{-2} - 0.65\times10^{-2}} \approx 0.1 \quad \text{[s]}$$

Very short period !!
(same as shown in the example in p.1.12)

In general, the reactor should be operated under the prompt criticality.

ρ / β : reactivity in [dollar] unit ([dollar]/100 = [cent])

Appendix 2

Solution of kinetics equations with six-DN-group model (1/3)

As a next step, we solve the kinetics equations of six-DN-group model :

$$\frac{d\,N(t)}{d\,t} = \frac{\rho-\beta}{\Lambda} N(t) + \sum_{i=1,6} \lambda_i\, C_i(t), \qquad (1)$$

$$\frac{d\,C_i(t)}{d\,t} = \frac{\beta_i}{\Lambda} N(t) - \lambda_i\, C_i(t). \quad (i=1,6) \qquad (2)$$

To solve these differential equations, the **exponential form** for the numbers of both neutrons and precursors are assumed again.

$$N(t) = A_N \exp(\omega t) \qquad C_i(t) = A_{Ci} \exp(\omega t) \qquad (3)$$

Introducing these expressions to the kinetics equations, we arrive the **inhour equation** based on six-DN-group model (see appendix for derivation).

$$\rho = \omega\Lambda + \sum_{i=1,6} \frac{\omega}{\omega+\lambda_i}\beta_i\ , \qquad (4)$$

$$\rho = \frac{\omega l}{1+\omega l} + \frac{1}{1+\omega l}\sum_{i=1,6} \frac{\omega}{\omega+\lambda_i}\beta_i\ . \qquad (5)$$

Difference from one-DN-group approximation

Solution of ... with six-DN-group model (2/3)

$$\rho = \frac{\omega l}{1+\omega l} + \frac{1}{1+\omega l}\sum_{i=1,6}\frac{\omega}{\omega+\lambda_i}\beta_i \quad (5')$$

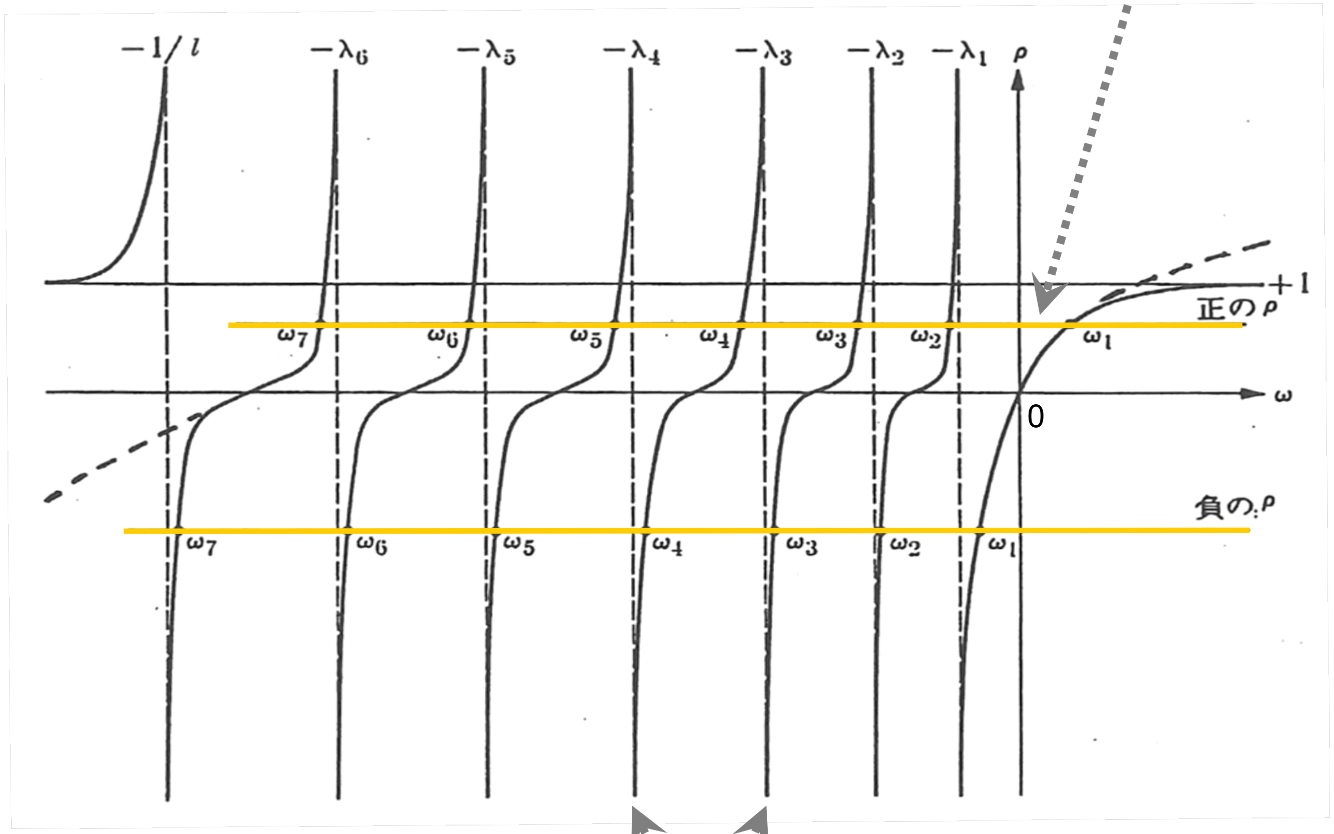
Horizontal line corresponds to the left hand side.

- **For 0 < ρ**

One positive root $\omega 1$ and six negative roots $\omega 2 \sim \omega 7$

- **For ρ < 0**

Seven negative roots "The minimum" root is the $\omega 1$ which corresponds to the longest and (negative) period.



Curves are trace of the right hand side.

Fig.4 Graphical solution of inhour equation

Solution of ... with six-DN-group model (3/3)

$$N(t)/N(0)=\sum_{j=1,7} R_j(\omega_j)\exp(\omega_j t) \qquad (6)$$

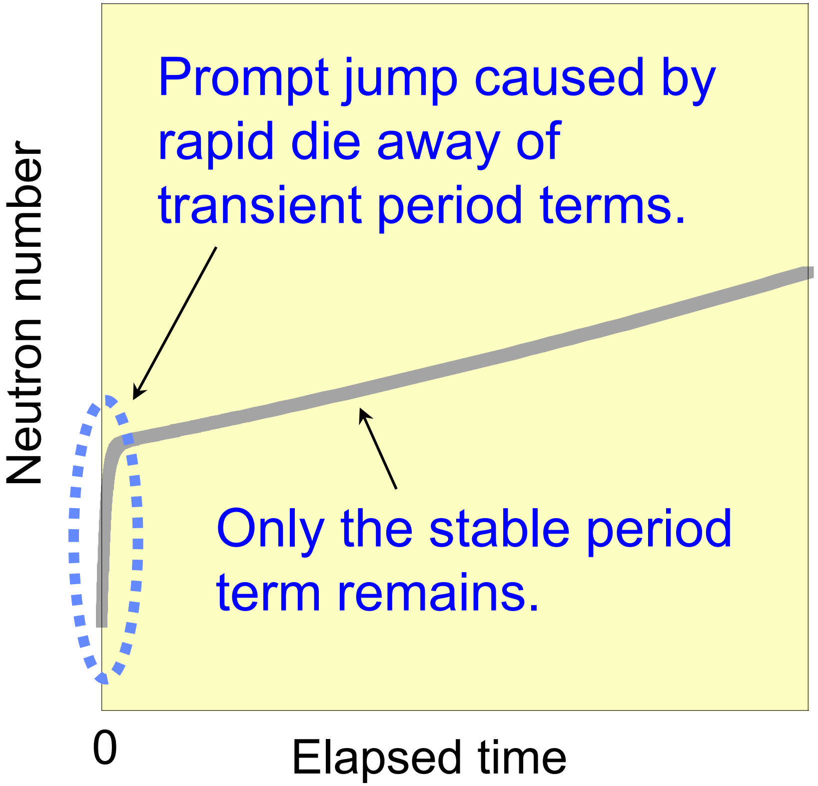
Linear combination of **seven exponential functions**

Amplitudes determined by initial conditions

$1/\omega_1$: stable period,

$1/\omega_2$ ~ $1/\omega_7$: six transient periods

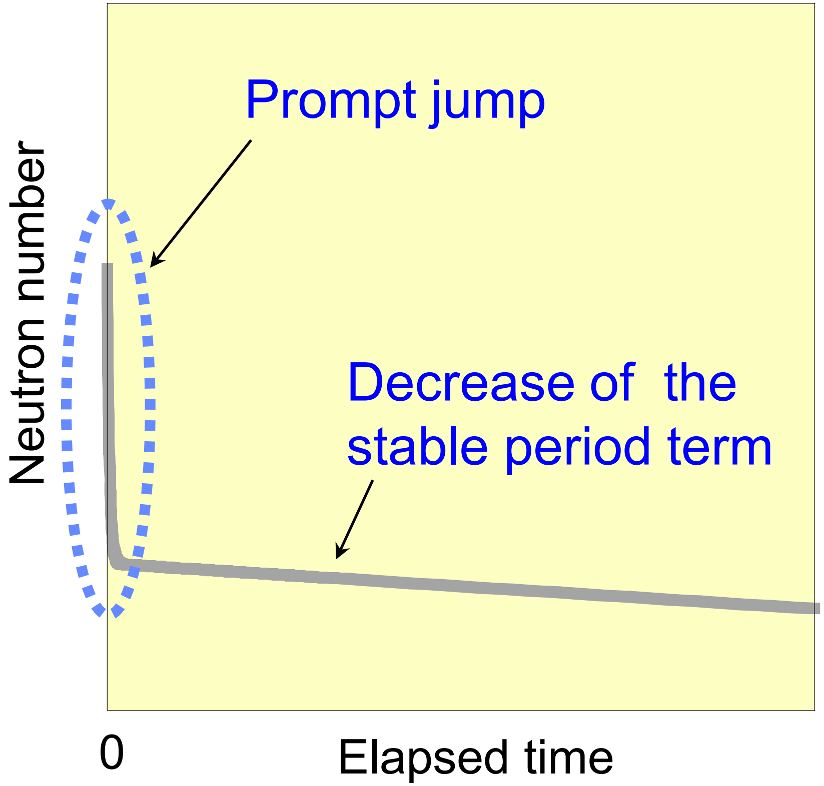
Positive reactivity insertion case



Prompt jump caused by rapid die away of transient period terms.

Only the stable period term remains.

Negative reactivity insertion case



Prompt jump

Decrease of the stable period term

Inhour equation (1/4)

From Eq.(4) of p.2.19 :

$$\rho = \omega \Lambda + \sum_{i=1,6} \frac{\omega}{\omega + \lambda_i} \beta_i \ ,$$

$$\Downarrow \quad \omega = 1/T$$

We arrive **another form of inhour equation with six-DN-group model :**

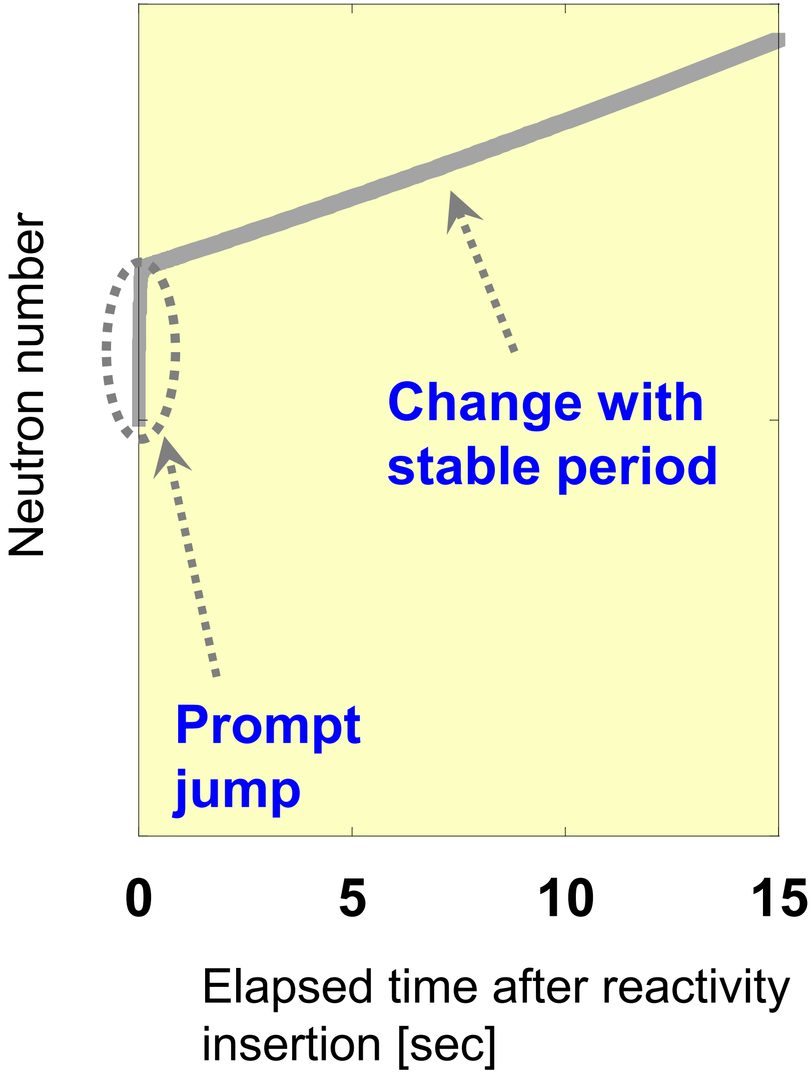
$$\rho = \frac{\Lambda}{T} + \sum_{i=1,6} \frac{\beta_i}{1 + \lambda_i T} . \qquad (1)$$

This is equivalent to Eq.(5) of p2.19 with the relation of $\omega = 1/T$.

The stable period is the longest root of this equation .

In the case of one-DN-group approximation,

$$\rho = \frac{\Lambda}{T} + \frac{\beta}{1 + \lambda T} . \qquad (2)$$

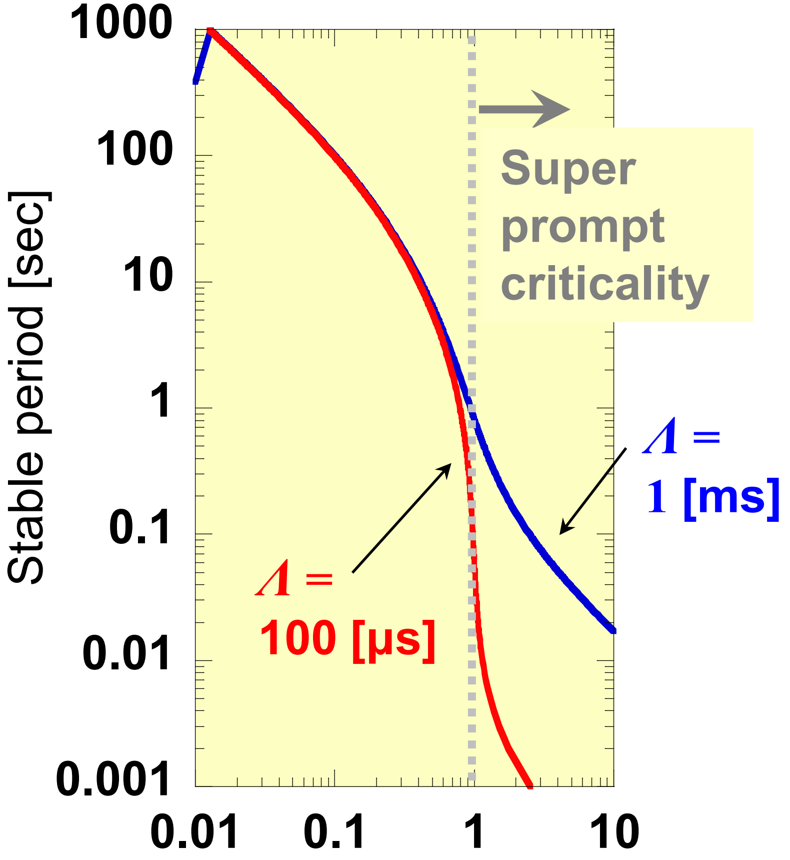


Inhour equation (2/4)

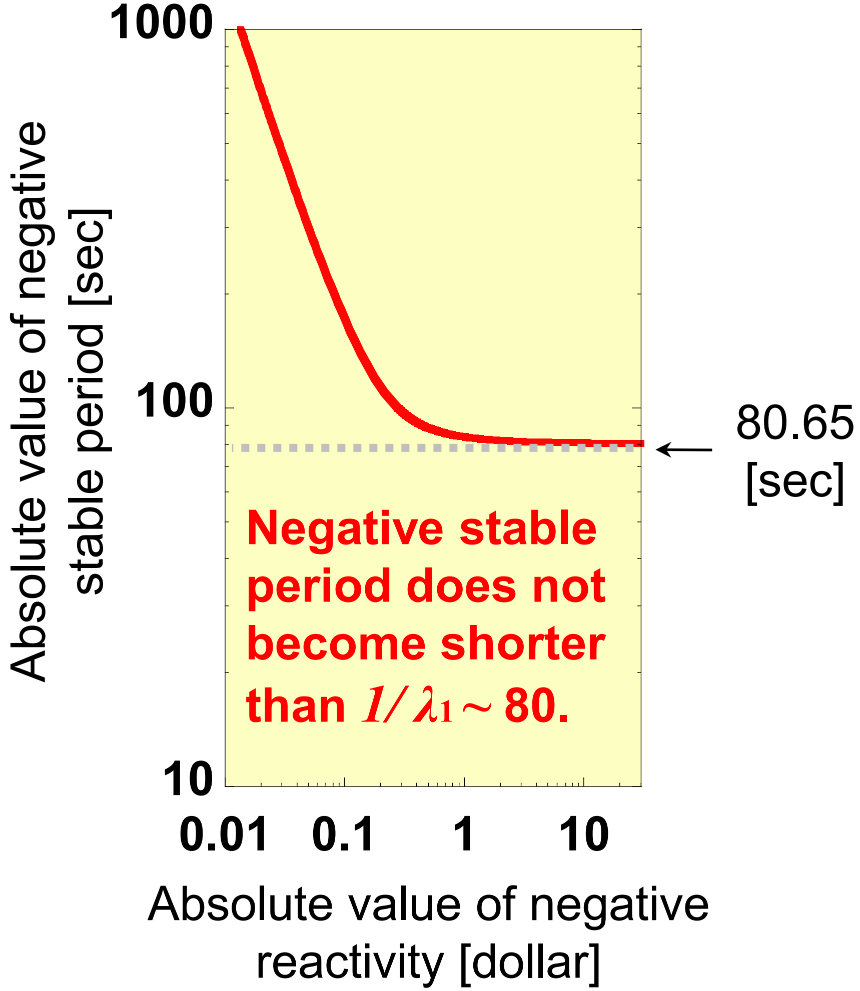
- Relation between inserted reactivity and stable period

Six-DN-group data of ^{235}U in p.1.5 are used.

Positive reactivity case



Negative reactivity case



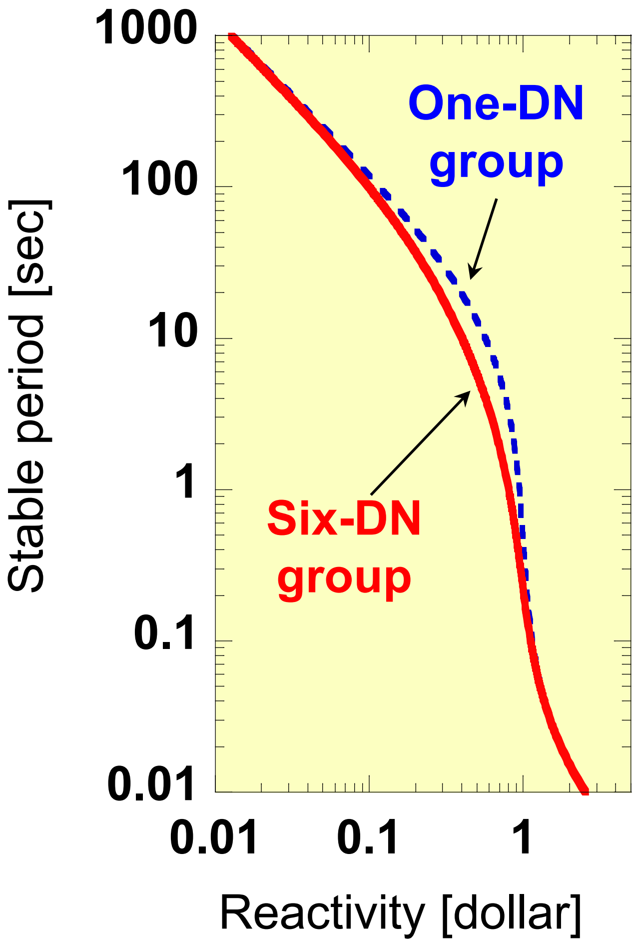
Negative stable period does not become shorter than $1/\lambda_1 \sim 80$.

Inhour equation (3/4)

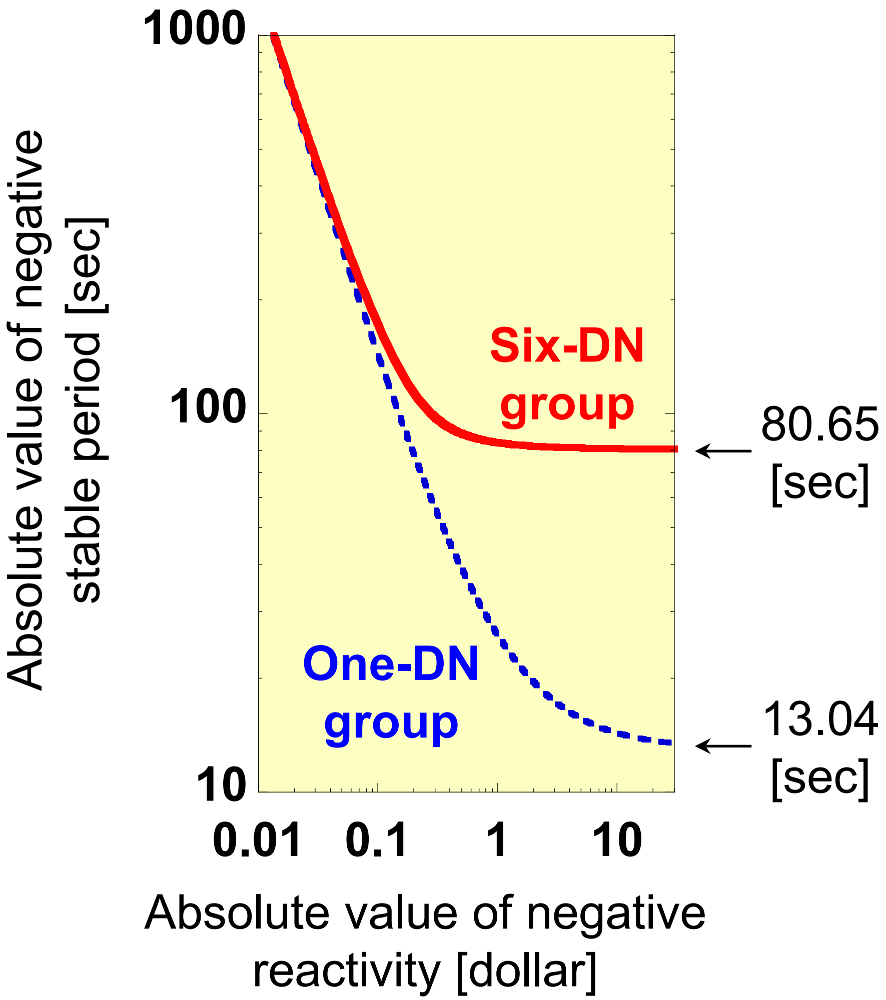
■ Comparison between one-DN and six-DN-group

One-DN and **six-DN**-group data of ^{235}U in p.1.5 are used.

Positive reactivity case



Negative reactivity case



Inhour equation (4/4)

■ Approximation to inhour equation to calculate stable period

- When $|\rho|<\beta$ and the stable period T is long :

$$\rho = \cancel{\frac{\Lambda}{T}} + \sum_{i=1,6}\frac{\beta_i}{1+\lambda_i T} \approx \sum_{i=1,6}\frac{\beta_i}{1+\lambda_i T} \qquad (3)$$

Solve this 6-th degree equation for the period.
The longest root is the stable period.

For a rough calculation, the one-DN-group approximation can be employed :

$$\rho \approx \frac{\beta}{1+\lambda T}, \quad \Rightarrow \quad T \approx \frac{\beta-\rho}{\lambda\,\rho}\left(=\frac{1}{\omega_+}\right). \qquad (4)$$

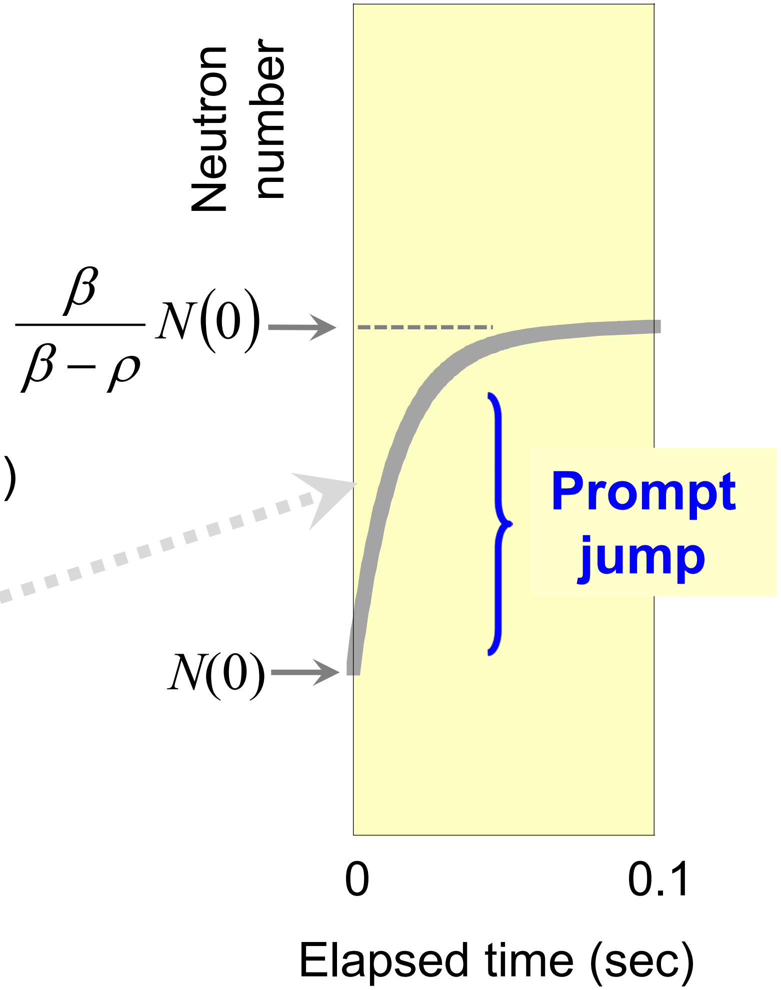
Reactor response Summary (2/3)

Short period around prompt jump(less than 0.1 [s]) after step reactivity insertion

Stays unity because of long period

$$N(t) \approx \frac{\beta}{\beta-\rho} N(0) \exp\left(\frac{\lambda\rho}{\beta-\rho} t\right) - \frac{\rho}{\beta-\rho} N(0) \exp\left(-\frac{\beta-\rho}{\Lambda} t\right) \quad (1')$$

$$N(t) \approx \frac{\beta}{\beta-\rho} N(0) - \frac{\rho}{\beta-\rho} N(0) \exp\left(-\frac{\beta-\rho}{\Lambda} t\right) \quad (2)$$



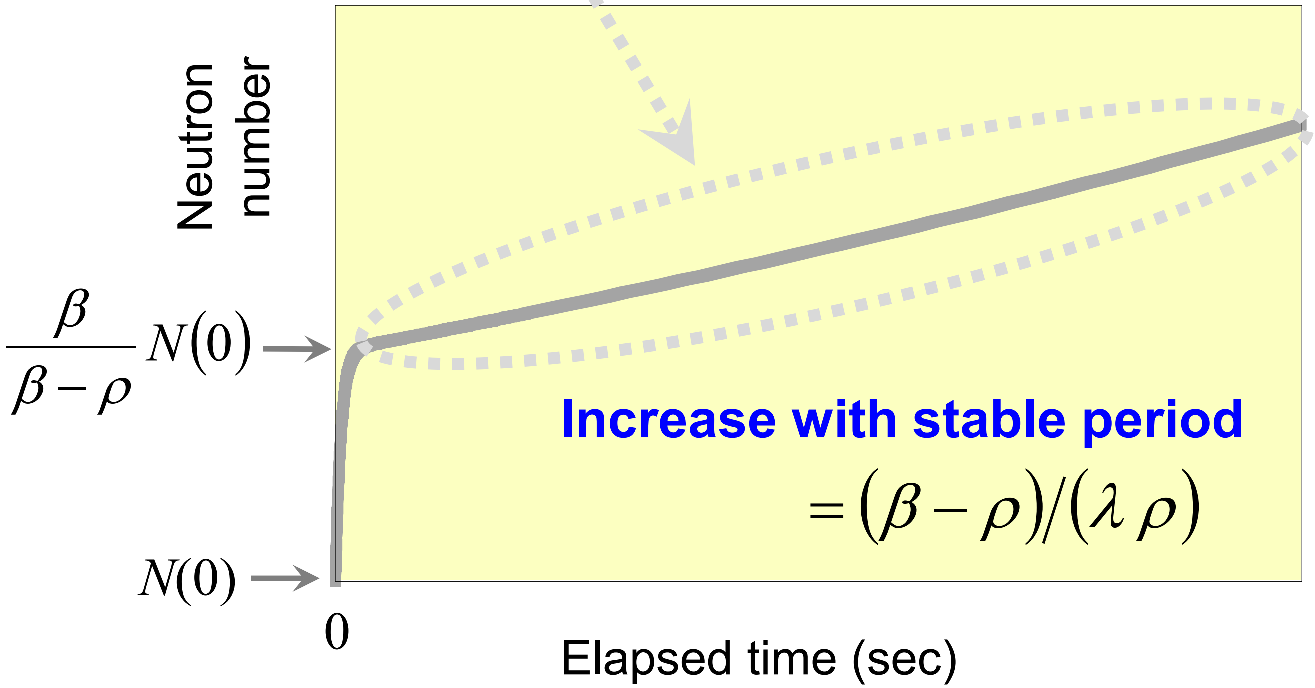
Reactor response Summary (3/3)

After prompt jump

$$N(t) \approx \frac{\beta}{\beta-\rho} N(0) \exp\left(\frac{\lambda\rho}{\beta-\rho} t\right) - \frac{\rho}{\beta-\rho} N(0) \exp\left(-\frac{\beta-\rho}{\Lambda} t\right) \quad (1')$$

Disappears because of negative and very short period

$$N(t) \approx \frac{\beta}{\beta-\rho} N(0) \exp\left(\frac{\lambda\rho}{\beta-\rho} t\right) \quad (3)$$



Increase with stable period

$$= (\beta-\rho)/(\lambda\rho)$$

Appendix A

Roots of characteristic equation of kinetics equation with one-DN-group approximation (Eq.(11) in p.2.6)

Derivation of roots of Eq(11) (1/2)

When $\rho << \beta$,

$$\left\{\begin{array}{lll} \dfrac{\beta-\rho}{\Lambda} \approx \dfrac{\beta}{\Lambda} \quad \dfrac{\beta}{\Lambda} >> \lambda & \Rightarrow & \dfrac{\beta-\rho}{\Lambda} >> \lambda \\ (\rho-\beta)^2 \approx \beta^2 \quad \beta^2 >> 4\,\lambda\,\rho\,\Lambda & \Rightarrow & (\rho-\beta)^2 >> 4\,\lambda\,\rho\,\Lambda \end{array}\right.$$

$$\omega^2 + \left(\frac{\beta-\rho}{\Lambda} + \cancel{\lambda}\right)\omega - \frac{\lambda\,\rho}{\Lambda} = 0 \quad \Rightarrow \quad \omega^2 + \left(\frac{\beta-\rho}{\Lambda}\right)\omega - \frac{\lambda\,\rho}{\Lambda} \approx 0$$

(8) in p2.5

The roots are given by

$$\omega_{\pm} = \frac{1}{2}\left\{\frac{\rho-\beta}{\Lambda} \pm \sqrt{\left(\frac{\rho-\beta}{\Lambda}\right)^2 + \frac{4\,\lambda\,\rho}{\Lambda}}\right\} = \frac{1}{2}\left\{\frac{\rho-\beta}{\Lambda} \pm \frac{|\rho-\beta|}{\Lambda}\sqrt{1+\frac{4\,\lambda\,\rho\,\Lambda}{(\rho-\beta)^2}}\right\} \qquad (1)$$

Furthermore,

$$\sqrt{1+\frac{4\,\lambda\,\rho\,\Lambda}{(\rho-\beta)^2}} \approx 1+\frac{1}{2}\frac{4\,\lambda\,\rho\,\Lambda}{(\rho-\beta)^2} = 1+\frac{2\,\lambda\,\rho\,\Lambda}{(\rho-\beta)^2} \qquad (2)$$

$$4\,\lambda\,\rho\,\Lambda/(\rho-\beta)^2 << 1 \quad \Leftarrow \quad (\rho-\beta)^2 >> 4\,\lambda\,\rho\,\Lambda$$

Derivation of roots of Eq(11) (2/2)

In the case of $\rho < \beta$, using the above approximation,

$$\omega_{\pm} \approx \frac{1}{2}\left\{\frac{\rho-\beta}{\Lambda} \mp \frac{\rho-\beta}{\Lambda}\left(1+\frac{2\,\lambda\,\rho\,\Lambda}{(\rho-\beta)^{2}}\right)\right\} \qquad (3)$$

Finally,

$$\omega_{+} \approx \frac{1}{2}\left\{\frac{\rho-\beta}{\Lambda} - \frac{\rho-\beta}{\Lambda}\left(1+\frac{2\,\lambda\,\rho\,\Lambda}{(\rho-\beta)^{2}}\right)\right\} = \frac{\lambda\,\rho}{\beta-\rho}, \qquad (4)$$

$$\omega_{-} \approx \frac{1}{2}\left\{\frac{\rho-\beta}{\Lambda} + \frac{\rho-\beta}{\Lambda}\left(1+\frac{2\,\lambda\,\rho\,\Lambda}{(\rho-\beta)^{2}}\right)\right\} = \frac{\rho-\beta}{\Lambda}\left(1+\cancel{\frac{\lambda\,\rho\,\Lambda}{(\rho-\beta)^{2}}}\right)$$

$$\approx -\frac{\beta-\rho}{\Lambda}. \qquad (5)$$

$$\frac{\lambda\,\rho\,\Lambda}{(\rho-\beta)^{2}} << 1$$

Appendix B

Derivation of inhour equation with six-DN-group model

The kinetics equation with six-DN-group model.

$$\begin{cases} \dfrac{d\,N(t)}{d\,t} = \dfrac{\rho-\beta}{\Lambda} N(t) + \sum\limits_{i=1,6} \lambda_i\, C_i(t), & (1) \\ \dfrac{d\,C_i(t)}{d\,t} = \dfrac{\beta_i}{\Lambda} N(t) - \lambda_i\, C_i(t). \quad (i=1,6) & (2) \end{cases}$$

To solve these differential equations, the exponential form for the numbers of neutrons and precursors are assumed again.

$$N(t) = A_N \exp(\omega t) \qquad C_i(t) = A_{Ci} \exp(\omega t) \qquad (3)$$

Introducing these expressions to the kinetics equations, we have

$$\begin{cases} \omega A_N = \dfrac{\rho-\beta}{\Lambda} A_N + \sum\limits_{i=1,6} \lambda_i\, A_{Ci}\,, & (4) \\ \omega A_{Ci} = \dfrac{\beta_i}{\Lambda} A_N - \lambda_i\, A_{Ci}\,. \end{cases}$$

➡

$$A_{Ci} = \frac{1}{\omega+\lambda_i} \frac{\beta_{eff}}{\Lambda} A_N \qquad (5)$$

From Eqs.(4) and (5) :

$$\left\{ \frac{\rho-\beta}{\Lambda} - \omega + \sum_{i=1,6} \frac{\lambda_i}{\omega+\lambda_i} \frac{\beta_i}{\Lambda} \right\} A_N = 0\,. \qquad (6)$$

The A_N should not always be 0, therefore

$$\frac{\rho-\beta}{\Lambda}-\omega+\sum_{i=1,6}\frac{\lambda_i}{\omega+\lambda_i}\frac{\beta_i}{\Lambda}=0\,. \qquad (7)$$

$\Downarrow$

$$\rho=\omega\Lambda+\beta-\sum_{i=1,6}\frac{\lambda_i}{\omega+\lambda_i}\beta_i \qquad (8)$$

$$=\omega\Lambda+\sum_{i=1,6}\frac{\omega+\lambda_i}{\omega+\lambda_i}\beta_i-\sum_{i=1,6}\frac{\lambda_i}{\omega+\lambda_i}\beta_i \qquad (9)$$

$$=\omega\Lambda+\sum_{i=1,6}\frac{\omega}{\omega+\lambda_i}\beta_i \qquad (10)$$

$$\left(\Lambda=l/k\right)$$

$$=\omega\frac{l}{k}+\sum_{i=1,6}\frac{\omega}{\omega+\lambda_i}\beta_i \qquad (11)$$

$$\left(1/k=1-\rho\right)$$

$$=\omega l\left(1-\rho\right)+\sum_{i=1,6}\frac{\omega}{\omega+\lambda_i}\beta_i \qquad (12)$$

$$\Rightarrow \quad \rho=\frac{\omega l}{1+\omega l}+\frac{1}{1+\omega l}\sum_{i=1,6}\frac{\omega}{\omega+\lambda_i}\beta_i \qquad (13)$$

Appendix C

General definition of reactor period

General definition of reactor period

Reactor period is generally defined as inverse of relative change rate of $N(t)$,

➡ $$T(t) \equiv \left(\frac{1}{N(t)}\frac{d\,N(t)}{d\,t}\right)^{-1} = \left(\frac{d\ln N(t)}{d\,t}\right)^{-1} \qquad (21)$$

✓ **After the prompt jump and transient period term completely died away ,**

$$N(t) \propto \exp(t/T)$$ ➡ $$\frac{1}{d\ln N(t)/d\,t} = T \qquad (52)$$

Compare the stable period calculated by Eq.(21) and transient period

Time changing rate of $N(t)$ after the the prompt jump finished.

$$\left|\frac{\Lambda}{\rho-\beta}\right| \ll \left|\frac{N(t)}{dN(t)/dt}\right| \qquad (53)$$

Transient period with one-DN-group approx.

Stable period

$$\frac{\beta-\rho}{\lambda\rho}$$

➡ $$\left|\frac{dN(t)}{dt}\right| \ll \left|\frac{\rho-\beta}{\Lambda}N(t)\right| \qquad (54)$$

⬇

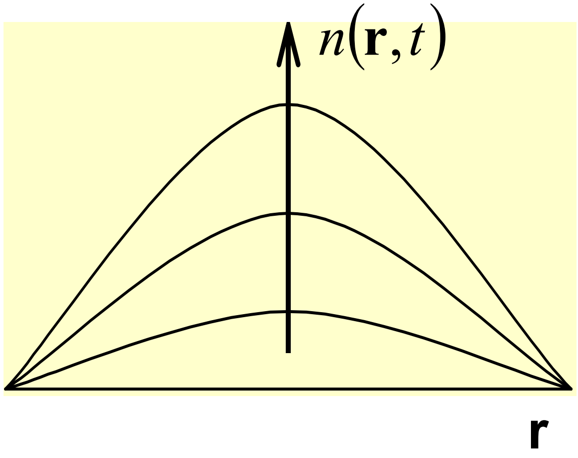
Use in the prompt jump approximation

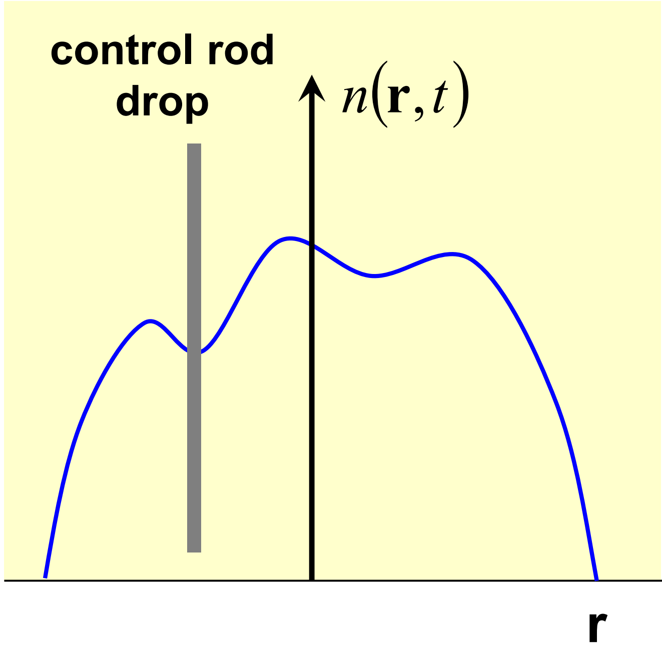
Appendix D

"One point" reactor kinetics equation

"One point" reactor kinetics equation

- During the derivation, we did not considered the spatial effects.
- The time-dependent behavior of the total neutron number is similar to that of neutron number density in any point in the reactor.
- It is therefore called **one point reactor kinetics equation**.
- For instance, large negative reactivity insertion with local absorber such as a control rod drop distorts the spatial distribution and induces the spatial effect that can not be neglected.
- In this case, the one-point approximation is less accurate and space - time kinetics is needed.
- Clearly one-point approximation has its limitations, but serves to illustrate the basic behavior of time-dependent reactor as discussed later.





Appendix E
Exercise

Question

We are operating a low-power research reactor. The scram power-level of this reactor is set at 120 [W].
The reactor power is so low that the temperature reactivity effect can be neglected. The reactor kinetics are treated by one-DN group approximation.

The kinetics parameters of this reactor are :

Neutron generation time Λ = 0.1 [ms]

Effective DN fraction β = 0.0068

DN precursor mean decay constant λ = 0.077 [s^{-1}]

This reactor is now kept critical at 100 [W]. Answer the following questions.

(1) A very small step reactivity of 0.03 [%Δk/k] is now inserted. Calculate a period from this reactivity insertion to the scram.

(2) A small step reactivity of 0.3 [%Δk/k] is now inserted. Choose the period from this reactivity insertion to the scram and show the reason of your choice.

(a) less than 1 [s] (b) 1 [s] ~ 100 [s] (c) more than 100 [s]